HOWARD'S

ANGLO-AMERICAN

Art of Reckoning

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C. Frusher Howard

(Fellow of the Society of Science, Letters and Art, London).

BY ORDER OF THE SCHOOL BOARD OF GLASGOW.

Text Book for Commercial Arithmetic, Advanced Evening Classes, High School.

HOWARD'S

ANGLO-AMERICAN

Art of Reckoning

THE STANDARD

TEACHER AND REFEREE

OF SHORTHAND

BUSINESS ARITHMETIC,

For the use of Schools and Business Colleges.

A Manual for the Counting House & Self Culture.

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INTRODUCTION.

"THE perfection of Art will be the most apt and efficient system of Rules."—*Karslake.*

"Every Science is evolved out of its corresponding Art." "There must be practice and an accruing experience, with its empirical generalisations before there can be Science." "Progress from the simple to the complex, from the concrete to the abstract, from the empirical to the rational."—*Herbert Spencer: Essay on Education.*

The ability to make business calculations with ease, accuracy, and speed, is an invaluable acquisition. The methods of Arithmetic used in schools are too tedious and complex for practical uses, they are weighted with superfluous elements that needlessly encumber the operations and distract and confuse the learner: the Rules in this New Art of Reckoning omit the needless work, and by an easily-learned, simple, and natural arrangement, lead directly to the required answer.

They are especially adapted to that large class of persons who find it difficult, or impossible, mentally to grasp, and retain complex numbers; such persons will find in this book

"A Complete Teacher of Business Arithmetic,"

all the examples being worked out, and explained so as to be readily understood, transforming the drudgery of calculation into a pleasing pastime, and qualifying persons of ordinary intellect to surpass the performances of the "Lightning Calculators" who have astonished mankind.

TO Accountants, Bankers, Brokers, Farmers, Merchants, Tradesmen, Practical Mechanics, &c., &c., a knowledge of the SCIENCE OF NUMBER is of minor importance; SKILL in the ART OF RECKONING is absolutely indispensable; the business of this book is by *new*, *original*, and *easily-acquired* methods, to teach that ART in accord with, yet distinct from, the SCIENCE; its province is to qualify the learner either for practical Business pursuits, or to study the science; he will master the higher Arithmetic with greater facility if he is *first* an *exact* and *Rapid Reckoner.*

The phenomenal success achieved by the former editions of this book—THREE HUNDRED AND SEVENTY thousand copies have been sold—encourages the hope that this new *Art of Reckoning* will soon be in every School, making ALL the boys "*Quick at Figures.*"

As a SCHOOL BOOK, its aim is to make the learner a GOOD CALCULATOR, with the greatest possible economy of time and study.

As a MANUAL for Business Men, Bookkeepers, Teachers, etc., to give the maximum of useful information in the briefest form consistent with clearness and completeness.

The REFERENCE TABLES are very comprehensive, and their arrangement simple and original.

The miscellaneous section is unique; it embraces almost every variety of BUSINESS CALCULATION, the work of finding the answer to each question is so expressed that it constitutes a formula for all similar examples.

One Reviewer of these Rules and Tables says:

"*Students, Teachers, and Business Men* can no more afford to be without them than they can afford to travel by OX-TEAMS, now the RAILWAY spans the Continent."

"Exact! Clear! Brief! Brilliant!"

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HOWARD'S
ART OF RECKONING.

DEFINITIONS AND SIGNS.

ART is knowledge *utilized.* SCIENCE is knowledge *organized.*

ART is *practical.* SCIENCE is *theoretical.*

ARITHMETIC is the ART OF RECKONING and the SCIENCE of number.

ACCOUNT SALES is a written account of goods sold, their price, expenses, and the net proceeds.

AGENT. An Agent, or Commission Merchant, transacts business for another person, who is called the *Principal.*

ANGLE, the difference in the direction of two lines proceeding from a common point; also a corner, or point where two lines meet; a square corner is called a *right angle;* an angle greater or less than a right angle is called an *oblique angle.*

AREA, the surface included within any given lines.

ASSESSMENT, a specific sum charged against each share of a stock company, or against property for the purpose of Taxation.

ASSETS. the available property of a Person, or a Company.

Note. These definitions are limited to the sense in which the words are used in Practical Business Arithmetic.

For Definitions omitted here see related subjects in the Book.

ARITHMETICAL SIGNS are characters indicating operations to be performed; they are indispensable for briefly and clearly stating a problem:

$+$, *plus*, and or more, signifying addition;

$-$, *minus*, less, signifying subtraction;

$\times$, *multiplied by*, as $2 \times 2 = 4$;

$\div$ signifies Division, or divided by; $6 \div 3$, or $\frac{6}{3}$ means 6 divided by 3; $\frac{6 \times 2}{3} = 4$, means 6 multiplied by 2, divided by 3, is equal to 4.

$=$ *is equal to*, as $\overline{6+2} \times 2 = 16$, and is read thus, "6 plus 2, multiplied by 2, is equal to 16";

$\overline{\quad\quad}$ the vinculum; or ( ) the Parenthesis, are used to show that the numbers to which they are applied, are to be considered as one quantity, thus $\overline{(6 \times 4) + 3 \times 2} \div \overline{4+2}$, means, the *sum* of the products of 6 multiplied by 4, and 3 multiplied by 2 is to be divided by the *sum* of 4 and 2.

$\sqrt{9}$, sign of the square root, read "the square root of 9";

4^2, sign of the square, read "the square of 4";

$\sqrt[3]{8}$, the cube root of 8. 8^3, the cube of 8.

BASE, the side upon which a figure is supposed to stand; also the foundation of a calculation.

BROKER, a person who buys or sells stocks, bills of exchange, real estate, etc., for another on commission. BULLS are Brokers who aim to increase the price of stocks, etc.: *Bears* are the opposite of *Bulls*. A CALL is the privilege to *demand the delivery* of shares of Stock within a certain time and at a certain price agreed upon; the privilege *to deliver* shares of Stock within a certain time is called a *Put;* the sum of money deposited with a Broker by a Speculator in Stocks to secure the Broker against loss is called *Margin* or *Cover.*

Capital. Of the money, stock, or other property employed in a business that part which belongs to the Firm or Principal is called the *Capital.*

Commission is the fee allowed an agent, usually at some rate per cent.

Consignment. Goods sent to an agent to be sold ; the person who sends the goods is called the *Consignor*, or *Shipper;* the person to whom they are sent is the *Consignee.*

Circle, a plane figure bounded by a single curved line, called its *circumference*, of which every part is equidistant from its centre.

Circumference, the line that bounds a Circle.

Cylinder, a straight round body, of uniform diameter, its ends being equal and parallel circles.

Cube, a solid body with six equal square sides. Also the product formed by the multiplication of three equal factors, as $4 \times 4 \times 4 = 64$, the *cube* of 4.

Cube Root. The Cube Root of any number is the number which when used three times as a factor produces that number by multiplication: thus, 4 is the Cube Root of 64, because $4 \times 4 \times 4 = 64$.

Currency, lawful money; coin or notes, or both ; gold and silver coins are sometimes called *Specie.* A currency whose denominations increase and diminish in a tenfold ratio is called *Decimal Currency.*

Diameter, a straight line passing from one side to the other through the centre of a circle, a sphere, &c.

DIVIDEND. A sum divided or to be divided, also The *pro rata* division of assets among creditors or profits among stockholders.

ELLIPSE, an oval or egg-shaped figure.

FACTORS. Numbers, the multiplication of which gives the product; thus 2, 5 and 6 are the factors of 60; the divisor and quotient of a number are its factors.

FIGURE. A figure is a sign representing a number.

FORMULA, a rule or principle concisely expressed by the use of signs.

GRAVITY, the force by which all bodies are attracted to the centre of the earth.

INSOLVENT, unable to pay.

INTEREST is the price or sum charged for the use of money. The sum of money bearing Interest, or invested, is called the *Principal*. *Simple Interest* is that which arises from the principal sum only. The sum of Principal and Interest is called the *Amount*.

INVENTORY, a catalogue of property on hand.

INVOICE, an account in detail of goods sold.

MANIFEST, a detailed list of a ship's cargo.

MATHEMATICS is the science of quantities.

MENSURATION is the art of measuring; *lineal* measure relates to length only; *superficial* measure to length and breadth; *cubic* or *solid* measure to length, breadth, and thickness.

MULTIPLE, a quantity which contains another a certain number of times without a remainder. A *common multiple* of two or more numbers is exactly divisible by each of them. The *least common multiple* is the *least* number that is so divisible; therefore, 30 is the *least common multiple* of 3, 5 and 6.

MOMENTUM, the quantity of motion in a moving body; the product of weight by velocity.

NET PROCEEDS, the sum remaining after all expenses are paid.

NUMBER, that which expresses quantity of units; a *Whole Number*, or *Integer*, is a number of whole units. A *Mixed Number* consists of an integer and a fraction, as $7\frac{3}{8}$, etc. A *Prime Number* is one that cannot be separated into two or more integral factors; an *abstract number* is a number used without reference to any particular object, as 9, 184, etc. A number used with reference to real things is called a *Concrete Number*.

PER CENT., from *per centum*, by the hundred; any per cent. of a number is therefore so many hundredths of that number; one per cent. is one of each hundred; the number per hundred taken is called the *Rate* per cent.

PER CENTAGE, the Per Centage is the sum total obtained by taking any specified number of hundredths of any given number.

POWER, a *power* is the product arising from multiplying a number by itself, or repeating it several times as a factor; thus, $3 \times 3 \times 3$, the product, 27, is the third power of 3.

The *Exponent* of a *Power* is the number denoting how many times the factor is repeated to produce the *power*, and is written thus: 2^2, 2^3.

$2 \times 2 = 2^2 = 4$, the second *power* of 2.
$2 \times 2 \times 2 = 2^3 = 8$, the third *power* of 2.

PYRAMID, a solid body uniformly tapering to a point at the top, standing on a plane, or flat surface, the base having straight sides; if the base is round, the body is called a *Cone;* the part that remains of a Pyramid, or of a Cone, after cutting off the top parallel with the base, is called its *Frustrum.*

QUADRANGLE, a plane figure with four angles, and consequently four sides.

QUANTITY, *how-much-ness*, or the expression of measurements. Anything that can be measured.

RADIUS, half the diameter of a circle. A straight line passing from the centre to the circumference.

RECIPROCAL is unity divided by a number. The *reciprocal* of any number or fraction is, therefore, that number or fraction inverted; thus the *reciprocal* of $\frac{4}{1}$ is $\frac{1}{4}$, of $\frac{3}{4}$ is $\frac{4}{3}$, of $3\frac{1}{3}$ is $\frac{3}{10}$.

RECTANGLE, a plane figure bounded by four sides, having all its angles right angles.

REDUCTION is the changing of quantities from one denomination to another without altering their value, as the reduction of fractions to other terms, the reduction of acres to yards, bushels to gallons, etc., etc.

RULE, a *rule* is a prescribed method of performing an operation.

SCALE, a scale is a series of numbers regularly ascending or descending.

A SOLID or BODY has length, breadth and thickness.

SPHERE, a body in which every part of the surface is equally distant from the centre.

SURFACE or SUPERFICES, the exterior face of anything.

SQUARE, a figure having four equal sides, and four right angles. The product of a number multiplied by itself; thus 16 is the square of 4.

SQUARE ROOT is the number, which multiplied into itself, produces the number of which it is the *root.* 4 is the *square root* of 16. $4 \times 4 = 16$.

TERMS, the terms of a fraction are numerator and denominator taken together.

The terms of a Proportion are its members.

TRIANGLE, a figure with three sides and three angles.

UNIT. A unit is *the* one; one taken in the abstract is called an *Abstract Unit,* in distinction from a concrete or Denominate Unit. A Fractional Unit is the unit of a fraction; thus $\frac{1}{4}$ is the unit of $\frac{3}{4}$.

UNITY, any definite quantity taken as *the* one.

USURY is a higher rate of interest than is allowed by law.

VALUE is the estimated price or equivalent of anything; in trading, Money is the measure of value.

VOLUME is the solid contents of a body; the space included by the surfaces that bound it.

WEIGHT is the measure of gravity.

ZERO, 0, naught, the starting point of a scale or reckoning.

NOTATION is the act of expressing numbers by figures.

All numbers are represented by the ten following figures: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

To establish their significance clearly in the minds of beginners it will be of great advantage occasionally to write and read them in the following manner:

$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{5}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$	$\frac{9}{1}$	$\frac{0}{1}$
one-one	two-ones	three-ones	four-ones	five-ones	six-ones	seven-ones	eight-ones	nine-ones	no-ones.

Figures have *simple* values and *local* values.

The *simple* value of a figure is the value it expresses when it stands alone.

The *local* value of a figure is the increased value which it expresses by having other figures placed on its right.

Each removal of a figure one place to the left increases its value ten times.

NUMERATION is reading numbers expressed by figures:

Tredecill'ns, Duodecill'ns, Undecillions, Decillions, Nonillions, Octillions, Septillions, Sextillions, Quintillions, Quadrillions, Trillions, Billions, Millions, Thousands, Units.

121,227,196,497,321,415,716,219,304,196,218,316,415,207,126.

Sex. Quin. Quad. Tril. Bill. Mill.

To read numbers expressed by figures: Point them off into periods of three figures each, commencing at the right hand; then, beginning at the left hand, read the figures of each period in the same manner as those of the right hand period are read, and at the end of each period pronounce its name.

NOTE.—By the English method of numeration the periods from millions upward have the same name, but consist of six figures each. What is called a Billion in the British Empire is elsewhere called a Trillion.

ADDITION.

ADDITION is the adding of numbers.

The answer is called the *Sum.*

Various suggestions have been made referring to improved methods of addition. In nearly every case the proposed improvement has been more fanciful than real. In practice, I have found no better or quicker method than the following:

```
 3746
 8743
 6978
 1256
 3021
 ----
23744
```

Commence at the bottom of the right hand column; add thus, 7, 15, 18, 24; set down the 4 in unit's place, and carry the two tens to the second column; then add thus, 4, 9, 16, 24; set down the 4 in ten's place, and carry the two hundreds to the third column, and so on to the end. Never add in this manner: 1 and 6 are seven, and 8 are 15, and 3 are 18, and 6 are 24. It is just as easy to name the *sum* at once, omitting the name of each separate figure, and saves two thirds of time and labor.

Book-keepers and others who have long columns of figures to add will find the following methods and suggestions acceptable.

8
4
7^2
9
6
5
8
7
9
4
6
8
7
9
8
9

In adding long columns of figures, write in the margin, lightly with pencil, opposite the last figure added, the unit figure of the sum immediately exceeding 100. By doing this the mind is never burdened with numbers beyond 100; and if interrupted in the work, it can be resumed at the stage at which the interruption occurred. The example in the margin shows the method; the small figure 2 on the right of the 7 shows that the *sum* of the column so far, the 7 inclusive, is 102.

INSTANTANEOUS ADDITION BY COMBINATION.

Write two, three, four, or more rows of miscellaneous figures, then write such figures as will make an equal number of nines in each column; under these again, write another row of miscellaneous figures.

EXAMPLE

```
  4 9 8 7
  4 7 3 6
  2 1 8 7
  5 0 1 2 one 9.
  5 2 6 3 two 9's.
  7 8 1 2 three 9's.
  4 9 8 6
 ---------
 3 4 9 8 3*
```

RULE. Bring down the last row, less the number of nines in each column, and prefix the number of nines.

* This addition by combination is introduced here merely as an amusing exercise. The nines are the result of combining the figures in the 1st and 4th, the 2nd and 5th, and the 3rd and 6th lines.

Rule of addition for two columns at once: first practice adding two columns of two figures each, until you are able to grasp at a glance, and pronounce their sum.

23	15	32	38	87	56
14	33	44	57	41	78
37	48	76	95	128	134

Add from the left, and say *three seven*, *four eight*, *twelve eight*, &c., &c., instead of *thirty-seven*, *forty-eight*, *one hundred and twenty-eight*, &c., &c.; this habit is readily acquired and saves half the time.

When you can instantly, at sight, name the sum of two pairs of figures, practice with gradually increasing columns of pairs, then take examples consisting of two or more columns of pairs.

	36			2147
	41			3472
47	74		*	1463
83	22		4614	2634
32	36	2123	7843	1785
21	41	4679	2183	6823
183	250	6802	14640	18324

* The process is *twelve six*, *one four* naught; the 40 is put down and the 1 carried to the units column in the next pair, then *ten naught*, *one four six*.

Any person who will PRACTICE this method, may add two columns with perfect ease; there is no royal road to this accomplishment: speed with precision can be attained only by persistent PRACTICE.

Fives are always easy to add; so are 9's, when it is borne in mind that adding 9 to a sum places it in the next higher ten with the unit 1 less; thus, $17 + 9 = 26$; $39 + 9 = 48$; $63 + 9 = 72$.

SUBTRACTION

is the process of finding the difference of two numbers by taking one number called the *Subtrahend* from another number called the *Minuend*.
The answer is called the *Difference or Remainder*.

RULE. Write the numbers so that the units in the subtrahend shall be directly under the units of the same order in the minuend; under, and in the same order, write the difference.

Subtract 473 from 1694.

$$\begin{array}{r} 1694 \\ 473 \\ \hline 1221 \end{array}$$

To prove Subtraction, add the *difference* to the *subtrahend*; if correct, their sum = the *minuend*.

MULTIPLICATION.

MULTIPLICATION is a shorter method of finding the *Sum* that is found by taking, or by repeating and adding, one number called the *Multiplicand*, as many times—or parts of a time—as there are units in another number called the *Multiplier*. The answer is called the *Product*.

Note. The multiplier must be an abstract number.

The base of our system of notation is 10; therefore numbers increase and diminish in a tenfold ratio; increasing from the decimal point to the left, and decreasing from the decimal point to the right; hence to multiply any number by 10, annex a cipher, or remove the point one place to the right. To multiply any number by 100, annex two ciphers, or remove the point two places to the right. To multiply any number by 1000, annex three ciphers, or remove the point three places to the right.

In multiplying be careful always to write the units, tens, etc., of the multiplier under the units, tens, etc., of the multiplicand, and the figures of the product in the same order.

To find the product of two numbers, each expressed by two figures only.

Multiply 54 by 32.

$$\begin{array}{r} 54 \\ 32 \\ \hline 1728 \end{array}$$

Process. First multiply the units figure of the multiplicand by the units figure of the multiplier, thus: $4 \times 2 = 8$; put the 8 in the units place in the product, then $\overline{5\times2} + \overline{4\times3}=22$, put the units 2 on the left of the 8 and carry the other 2; then, $\overline{5\times3}+2=17$, which, put down, making a total of 1728, the answer.

The same method can be applied when the multiplicand has three or more figures.

$$\begin{array}{r} 163 \\ 24 \\ \hline 3912 \end{array}$$

The steps are: $3 \times 4 = 12$, set down the 2 and carry the 1; $\overline{(6 \times 4)} + \overline{(3 \times 2)} + 1 = 31$; set down the 1, and carry the 3. $\overline{(1 \times 4)} + \overline{(6 \times 2)} + 3 = 19$; set down 9 and carry 1; $1 \times 2 + 1 = 3$, which place at the head of the line, making a total of 3912.

When the multiplier can be resolved into two factors, it is sometimes shorter to multiply by each factor, than by the whole number.

Example, multiply 163 by 24.
$8 \times 3 = 24$.

$$\begin{array}{r} 163 \\ 8 \\ \hline 1304 \\ 3 \\ \hline 3912 \text{ Ans.} \end{array}$$

When either the tens or the units are alike.

RULE. Multiply the units, set down the unit figure of the product; multiply the sum of the unlike figures by one of the like figures, then multiply the tens figures together, adding the carrying figures as you proceed.

Multiply 92 by 97 and 74 by 24.

$$\begin{array}{r} 97 \\ 92 \\ \hline 8924 \end{array} \qquad\qquad \begin{array}{r} 74 \\ 24 \\ \hline 1776 \end{array}$$

When the units are alike and the sum of the tens is ten.

RULE. Add one of the units to the product of the tens, and annex the product of the units.

Multiply 74 by 34.

$$\overline{7\times3}+4 \text{ with } 16 \text{ annexed}=2516.$$

To multiply any two numbers between 10 *and* 20.

RULE. Add one number to the units of the other; call the sum tens, and add the product of the units.

$$18\times14=\overline{18+4} \text{ tens}+\overline{8\times4}=252.$$

The Area of a Circle=the square of the diameter $\times .7854$.

The following was published by Mr. J. Macfarlane Gray, in 1852, in a Magazine article :—

To multiply any number by .7854.

RULE. Multiply by 7, repeat, double and repeat, writing each successive product one place to the right.

$$\begin{array}{ll} .7 & =\text{the product of } 1\times.7. \\ \ \ 7 & =\text{repeat one place to the right.} \\ \ \ 14 & =\text{double " " " "} \\ \ \ \ 14 & =\text{repeat " " " "} \\ \hline .7854 & =1\times.7854. \end{array}$$

When the multiplier is any number between 11 and 20, the process is simply to multiply by the unit of the multiplier, set down the product under, and one place to the right *of*, and then add *to* the *multiplicand;* or multiply units by units, and then add to each succeeding product, the next figure to the right of the figure multiplied, and the figure carried.

EXAMPLE, multiply 1496 by 17.

$$\begin{array}{r} 1\,4\,9\,6 \\ 1\,0\,4\,7\,2 \\ \hline 2\,5\,4\,3\,2 \end{array}\text{. Ans.} \qquad \text{or thus:} \qquad \begin{array}{r} 1\,4\,9\,6 \\ 1\,7 \\ \hline 2\,5\,4\,3\,2 \end{array}$$

The process in the last example is:

$6 \times 7 = 42$, set down 2 and carry 4.
$9 \times 7 + 6 + 4 = 73$; carry 7.
$4 \times 7 + 9 + 7 = 44$; carry 4.
$1 \times 7 + 4 + 4 = 15$; carry 1.
$1 + 1 = 2$.

To multiply two figures by 11.

RULE. Between the two figures write their sum: thus: multiply 43 by 11. Ans. 473. The sum of 4 and 3 is 7; place the seven between the 4 and 3, for the product.

NOTE. Add one to the hundreds when the sum exceeds 9.

To multiply any number by 11.

RULE Bring down the extreme right hand figure, then add the right hand figure to the next, and bring down the sum; then add the second figure to the third and bring down the sum, adding in the figure carried, in each case, and so on to the end.

EXAMPLE $12345678 \times 11 = 135802458$.

To multiply any two numbers ending with 5.

RULE. Add $\frac{1}{2}$ the sum of the figures preceding the 5 in each number to the product of the same figures, and annex 25.

NOTE. When the sum of the preceding figures is an odd number, add half the number next smaller than the sum and annex 75.

Multiply 85 by 65 and 105 by 35.

$85 \times 65 = 7 + \overline{8 \times 6}$ with 25 annexed $= 5525$

$105 \times 35 = 6 + \overline{10 \times 3}$ " 75 " $= 3675$

To multiply when the unit figures added, equal 10, *and the tens are alike, as* 67 × 63.

RULE. Multiply the units and set down the result, then add one to the upper number in tens place, and multiply by the lower.

$$\begin{array}{r} 67 \\ 63 \\ \hline 4221 \end{array}$$

To multiply two numbers when either has one or more ciphers on the right, as 26 *by* 20, 244 *by* 200, *etc.*

RULE. Take the cipher or ciphers from one number and annex it, or them, to the other, multiply by the number expressed by the remaining figures.

EXAMPLE 1. Multiply 26 by 20. Ans. 520.

Process. 260 × 2 = 520.

2. Multiply 244 by 200. Ans. 48800.

24400 × 2 = 48800.

To multiply unlike numbers greater than a common base.

RULE. To the common base add the differences; multiply the sum by the base and add the product of the differences.

EXAMPLE. Multiply 603 by 612

$$\overline{603+12}\times600+\overline{3\times12}=369,036.$$

To multiply unlike numbers less than a common base.

RULE. To the multiplicand add the tens and units of the multiplier, less the last 1 to carry, multiply the sum by the common base and add the product of the differences.

EXAMPLE. Multiply 93 by 89 and 293 by 289.

$$\begin{array}{l} 89 \\ \underline{93} \\ 8277 \end{array} \qquad \begin{array}{l} 293 \\ \underline{89} \\ 282\times300+\overline{11\times7}=84,677. \end{array}$$

The product of any two numbers = the square of their mean, diminished by the square of half their difference. Multiply 22 by 18. $20^2-2^2=396$.

To multiply by 9, 99, 999, *&c.*

Annex as many ciphers as there are nines in the multiplier, and subtract the multiplicand.

$$358\times9=3580-358=3222.$$

To multiply two numbers having a common base, one ending with 25, *the other ending with* 75.

RULE. Multiply the common base by one more than itself and annex 1875.

EXAMPLE. Multiply 675 by 625.

6×7 with 1875 annexed = 421,875.

RAPID METHOD OF SQUARING NUMBERS.

BY THE DIFFERENCE OF A NUMBER AND ITS BASE.

To square a number greater than its base.

RULE. *Add to the given number the difference of that number and its base, multiply the sum by the base; to the product add the square of the difference.*

NOTE. Take the nearest convenient multiple of ten for the base.

EXAMPLE 1. What is the square of 11? Ans. 121.

Process. Taking 10 for the base, the difference is one $\overline{(1 + 11) \times 10} + 1^2 = 121.$

$(22)^2 = 484.$ Taking 20 for the Base the difference is *two.*
$\overline{(22+2) \times 20} + 2^2 = 484.$

$(104)^2 = 10{,}816.$ $\overline{(104+4) \times 100} + (4)^2 = 10{,}816.$

$(322)^2 = 103{,}684.$ $\overline{(322+22) \times 300} + (22)^2 = 103{,}684.$

$(813)^2 = 660{,}969.$ $\overline{(813+13) \times 800} + (13)^2 = 660{,}969.$

For squaring numbers less than the base.

RULE. From the number to be squared *subtract* the difference, *multiply* the result by the base, to the product *add* the square of the difference.

$(9)^2 = 81.$ Taking 10 for the Base the difference is *one.*
$\overline{(9-1) \times 10} + (1)^2 = 81.$

$(96)^2 = 9216;$ $\overline{(96-4) \times 100} + (4)^2 = 9216.$

$(27)^2 = 729.$ $\overline{(27-3) \times 30} + (3)^2 = 729.$

$\overline{(99{,}946-54) \times 100{,}000} + (54)^2 = 9{,}989{,}202{,}916 = (99{,}946)^2.$

Multiply £19,,19,,11$\frac{3}{4}$d by $19 + \frac{19}{20} + \frac{11}{240} + \frac{3}{960}.$

$\overline{(£19,,19,,11\frac{3}{4} - £\frac{1}{960}) \times 20} + £(\frac{1}{960})^2 = £399\frac{19}{20}\ \frac{2}{240}\ \frac{1}{921600}$

or $\overline{£19,,19,,11\frac{3}{4} \times 20} - \frac{1}{960} = £399,,19,,2\ \frac{1}{960}$ of a farthing.

NOTE. In squaring numbers between 50 and 60, take 50 for the base; to 25 add the difference, call the sum hundreds, to this add the square of the difference.

1. $(51)^2 = 2601.$

Process. $\overline{(25+1)\times 100}+1^2=2601.$

2. $(52)^2 = 2704.$

NOTE. In squaring numbers between 40 and 50; to 15 add the unit figure, call the number hundreds, to the sum add the square of the difference, taking 50 for the base.

1. $(41)^2 = 1681.$

Process. $\overline{(15+1)\times 100}+9^2=1681.$

2. $(42)^2 = 1764.$

By this rule the squares of all numbers up to 1000, and larger numbers near the multiples of 10 may be found with less labor than is required to find them in tables;

The square of any number ending with 25=half the number of hundreds + the square of the number of hundreds×10,000+625.

$$\overline{(3+6^2)\times 10{,}000}+25^2=390{,}625=625^2$$

It will sometimes be convenient to divide large numbers into two parts, and use the following formula:

"The square of any number = the sum of the squares of its two parts, plus twice the product of one part multiplied by the other part."

EXAMPLE. Find the square of 823,732

$$\begin{array}{rr} 823{,}000^2= & 677{,}329{,}000{,}000 \\ 823{,}000\times 732\times 2= & 1{,}204{,}872{,}000 \\ 732^2= & 535{,}824 \\ \hline & 678{,}534{,}407{,}824 \end{array}$$

NOTE.—Until this rule is thoroughly understood, the learner should limit his exercises to numbers near 10, 100, 1000, etc.; and then operate with more complex numbers.

When the multiplier is a number near, and less, than a multiple of 10.

RULE. Annex to the multiplicand as many ciphers as there are in the next order of tens higher than the multiplier, subtract the product of the multiplicand by the complement.

Multiply 222 by 93.

$$22{,}200-\overline{222\times7}=20{,}646.$$

When both numbers have a cipher in the tens place.

RULE. Write the product of the units, then the sum of the products of the upper hundreds by the lower units, and the lower hundreds by the upper units, prefix the product of the hundreds.

Multiply 409 by 704.

$$\begin{array}{r} 704 \\ 409 \\ \hline 287936 \end{array}$$

DIVISION.

Division is the process of finding how many times one number called the *Divisor* is contained in another number called the *Dividend.*

The answer is called the *Quotient.*

RULE To the left and in a line with the *dividend*, write the *divisor*, separated by an arc. Take so much of the dividend as contains a number less than ten times the *divisor;* the number of times the *divisor* is contained in that part of the *dividend* is the first figure in the quotient; annex the next unused figure of the *dividend* to the *remainder* to find the second figure of the quotient, and so on to the end.

Divide 49654809 by 4.

$$4\overline{)49654809}$$
$$\text{Ans. } 12413702\tfrac{1}{4}$$

Process The *divisor* 4 is contained in the first figure of the *dividend* once, therefore 1 is the first figure in the *quotient*: 4 is contained twice and 1 remainder in 9; 2 is then the second figure in the quotient: the next unused figure 6 annexed to the remainder 1=16: 4 is contained in 16 four times, and so on to the end.

Divide 7983204 by 23.

$$\begin{array}{r}
23)7983204(347095\frac{19}{23} \\
108 \\
163 \\
220 \\
134 \\
19
\end{array}$$

Process. $79-\overline{23\times3}$, the remainder is 10; the next unused figure in the *dividend* 8, annexed to 10=108; $108-\overline{23\times4}$, the remainder is 16; to this remainder annex the next unused figure in the dividend, and so on until the quotient is complete. When the divisoı is a composite number, divide by its factors.

Example. Divide 504 by 42. $42=7\times6$.

$$\begin{array}{r|l}
6 & 504 \\
7 & 84 \\
 & 12 \text{ Ans.}
\end{array}$$

FRACTIONS.

A Fraction is a part or parts of a unit, of a quantity, or of a whole number. *Common Fractions* are written with figures below the line called the *Denominator*, and figures above the line called the *Numerator*, thus $\frac{3}{4}$, three-fourths, $\frac{9}{5}$, nine-fifths, etc., the Denominator shows into how many parts *Unity* is divided; the Numerator shows how many parts are taken. When the numerator exceeds the denominator it is called an *Improper Fraction*.

Multiplying or *Dividing* both terms of a fraction by the same number does not change its value.

Multiplying the numerator, multiplies the fraction.

Dividing the numerator, divides the fraction.

Multiplying the denominator, divides the fraction.

Dividing the denominator, multiplies the fraction.

Fractions are called similar when they have a common denominator, as $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{5}$.

To find a common denominator for any two fractions, multiply both terms of either fraction by the denominator of the other; or if the denominator of one fraction will exactly divide the denominator of the other, multiply both terms of the one by the quotient.

To reduce a fraction to its simplest form.

Rule. Divide both terms by their greatest common divisor or its factors, the simplest form, or lowest term of $\frac{36}{48}$, is obtained by dividing both terms by 12, $\frac{36}{48} = \frac{3}{4}$.

To find the greatest common divisor of two numbers:

RULE. Divide the greater by the less, and the previous divisor by the remainder, and so on until there is no remainder; the last divisor is the answer.

Find the greatest common divisor of 18 and 27.

```
18)27(1
   18
    9)18(2
      18        Ans. 9.
```

To find the least common multiple:

RULE. Cancel all the numbers contained exactly in any of the others; divide those not canceled by a number that will exactly divide one or more of them; also divide each of the remaining numbers by the greatest factor common to it and the divisor; bring down each quotient with the undivided numbers, and proceed as before, until no two numbers have a common divisor; the product of the divisors and the remaining numbers is the answer.

Find the least common multiple of 24, 20, 18, 8, 9, 12, 15.

$$\begin{array}{l} 12)\ \cancel{24},\ \cancel{20},\ \cancel{18},\ \cancel{8},\ \cancel{9},\ \cancel{12},\ \cancel{15} \\ \quad\ \ \cancel{2},\ 10,\ \ 3 \qquad\qquad\quad \cancel{5} \quad 12\times10\times3=360.\ \text{Ans.} \end{array}$$

The number 12 has for factors 2, 3, 4, 6, 12, it is therefore often a good first divisor.

ADDITION OF FRACTIONS.

RULE. *Reduce the fractions to a common denominator; add the numerators and place the sum over the common denominator, or multiply either denominator by the other numerator, and place the sum of the products over the common denominator.*

add $\frac{2}{3}$ and $\frac{1}{4}$; $\frac{2}{3}=\frac{8}{12}$, $\frac{1}{4}=\frac{3}{12}$, $\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$, or

$3\times1+4\times2=11$ the numerator, $4\times3=12$ the denominator.

add $\frac{3}{5}$ and $\frac{1}{2}$, $\frac{3}{5}=\frac{6}{10}$, $\frac{1}{2}=\frac{5}{10}$, $\frac{6}{10}+\frac{5}{10}=\frac{11}{10}=1\frac{1}{10}$,

or $5\times1+2\times3=11$, the numerator. $5\times2=10$, the denominator.

SUBTRACTION OF FRACTIONS.

RULE. *Reduce the Fractions to a common denominator, and write the difference of the numerators over the common denominator.*

From $\frac{3}{4}$ take $\frac{1}{2}$. PROCESS: $\frac{1}{2}=\frac{2}{4}$; $\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$ Ans.
From $9\frac{1}{3}$ take $4\frac{1}{2}$. Ans. $4\frac{5}{6}$.
From $18\frac{3}{4}$ take $3\frac{1}{3}$. Ans. $15\frac{5}{12}$.

MULTIPLICATION OF FRACTIONS.

RULE. *Multiply the whole numbers together, then multiply the upper whole number by the lower fraction, and the lower whole number by the upper fraction; multiply the fractions together, and add all the products;* or reduce mixed numbers to improper fractions, and multiply the numerators by each other, and the denominators by each other, cancelling as shown on pages 40 and 42.

Multiply $9\frac{1}{3}$ by $2\frac{1}{4}$. Ans. 21.

$$\begin{array}{l} 9\frac{1}{3} \\ 2\frac{1}{4} \\ \hline 18 \\ 2\frac{3}{12}+\frac{8}{12}+\frac{1}{12}. \\ \hline 21 \end{array} \quad \text{or} \quad \begin{array}{l} 9\frac{1}{3}=\frac{28}{3} \\ 2\frac{1}{4}=\frac{9}{4} \end{array} \quad \frac{\overset{7}{\cancel{28}}\times\overset{3}{\cancel{9}}}{\cancel{3}\times\cancel{4}}=21.$$

MULTIPLICATION OF ENGLISH MONEY.

Multiply £9 7s. $5\frac{3}{4}$d. by 7.

£9	7s.	$5\frac{3}{4}$d.
		7
£65	12s.	$4\frac{1}{4}$d.

PROCESS. Say seven times $3=21$ farthings; put down $\frac{1}{4}$d. and carry 5d.; 7 times 5d. + 5d. = 3s. 4d.; put down 4d. and carry 3s.; 7 times 7s. + 3s. = £2 12s.; put down 12s. and carry £2; 7 times £9 + £2 = £65. Total £65 12s. $4\frac{1}{4}$d.

To find the cost of any quantity at any given number of shillings each.

Find the cost at 1 shilling and multiply by the price, or point off 1 place left and multiply by half the price.

What is the cost of 60 lbs. of tea @ 4s. per lb.?

60s.=£3 0s. 0d. £3 0s. 0d.×4=£12, or £6·0×2=£12 0s. 0d.

Find the cost of 49 yds. of cloth @ 7s. per yd.

£2 9s. 0d. × 7 = £17 3s. 0d., or £4·9 × $3\frac{1}{2}$ = £17 3s. 0d.

Find the cost of $47\frac{1}{4}$ lbs. of tea @ 5s. per lb.

£2 7s. 3d. × 5 = £11 16s. 3d., or £$47\frac{1}{4}$÷4 = £11 16s. 3d.

Find the cost of 190 bushels of wheat @ 4s. 4d.

£9 10s. 0d. × $4\frac{1}{3}$ = £41 3s. 4d., or £19·0 × $2\frac{1}{6}$ =£41 3s. 4d.

To multiply any two numbers together, ending with $\frac{1}{2}$, as $9\frac{1}{2}$ by $3\frac{1}{2}$.

RULE. To the product of the whole numbers, add half their sum, plus $\frac{1}{4}$.

NOTE. When the *sum* is an odd number take half the next number below it, and the fraction in the answer will be $\frac{3}{4}$.

1. What will $9\frac{1}{2}$ lbs. of rice cost, at $3\frac{1}{2}$ pence per lb? Ans. $33\frac{1}{4}$ pence.

Process. The sum of 9 and 3 is 12; half this sum is 6; then we say 9 times 3 is 27, and 6 = 33; to this add $\frac{1}{4}$.

To multiply any two numbers together having the same fraction.

RULE. To the product of the whole numbers, add the product of their sum by the fraction; to this add the product of the fractions.

1. What will $13\frac{3}{4}$ lbs. of beef cost, at $7\frac{3}{4}$ cents per lb? Ans. \$1.06$\frac{9}{16}$.

Process. The sum of 13 and 7 is 20, three-fourths of this sum is 15, so we say, 7 times 13 is 91, and 15 = 106, to which add the product of the fractions, ($\frac{9}{16}$) and the result is the Ans. \$1.06$\frac{9}{16}$.

In actual business calculations, any fraction *less* than a cent is reckoned as *one* cent; therefore in dealing with such questions as $13\frac{1}{3}$ pounds of beef at $7\frac{1}{5}$ cents a pound, it is sufficiently accurate to say:

$\frac{1}{5}$ of $13=3$. $\frac{1}{3}$ of $7=2$. $\overline{13\times7}+3+2=96$ cents;

Or $17\frac{1}{4}$ lbs. of cheese at $9\frac{1}{3}$ cents per pound.

$\frac{1}{3}$ of $17=6$. $\frac{1}{4}$ of $9=2$. $\overline{17\times9}+6+2=\$1.61$.

To multiply a fraction by an integer.

Multiply the numerator, or divide the denominator.

$\frac{3}{8}\times2=\frac{3}{4}$. $\frac{5}{9}\times3=\frac{5}{3}=1\frac{2}{3}$.

When the whole numbers are alike, and the sum of the fractions is a unit.

RULE. Take the *product* of the whole numbers, to this add the *integer* in the multiplicand, then add the *product* of the fractions, and the result will be the answer.

$2\frac{1}{2}\times2\frac{1}{2}=\overline{2\times2}+2+\overline{\frac{1}{2}\times\frac{1}{2}}=6\frac{1}{4}$.

$7\frac{7}{8}\times7\frac{1}{8}=\overline{7\times7}+7+\overline{\frac{7}{8}\times\frac{1}{8}}=56\frac{7}{64}$.

$96\frac{7}{9}\times96\frac{2}{9}=(96)^2+96+\frac{7}{9}\times\frac{2}{9}=9312\frac{14}{81}$.

$9956\frac{41}{44}\times9956\frac{3}{44}=(9956)^2+9956+\frac{41}{44}\times\frac{3}{44}=99,131,892\frac{123}{1936}$.

DIVISION OF FRACTIONS.

Rule: Reduce whole and mixed numbers to improper fractions, then multiply the numerator of the dividend by the denominator of the divisor and divide the product by the other two terms; or,

Reduce to a common denominator and divide the numerator of the dividend by the numerator of the divisor.

Divide $8\frac{1}{2}$ by $1\frac{1}{4}$. $8\frac{1}{2}=\frac{17}{2}$ $1\frac{1}{4}=\frac{5}{4}$ $\frac{17\times4}{2\times5}=6\frac{4}{5}$ Ans.

or $8\frac{1}{2}=\frac{34}{4}$. $1\frac{1}{4}=\frac{5}{4}$, $\frac{34}{5}=6\frac{4}{5}$. Ans.

See pages 40 and 42.

THE UNIT—or one thing—is the idea of number in its simplest form. UNITY is the basis of every number, the primary base of every fraction, the *unit* of *six* months is *one* month, the unit of a fraction is the reciprocal of the denominator, thus $\frac{1}{8}$ is the *unit* of $\frac{5}{8}$; every step from the unit increases the complexity of numbers, and consequently demands an increase of mental power and energy in dealing with them; when it happens that a reckoning requires the use of one or more multipliers and one or more divisors, subject the *unit* of value to all the required processes, and the answer will be a common multiplier for all like examples.

If lard is worth \$6·75 in New York per 100 lbs. what is the value in £ sterling of 112 lbs.: the £ being worth \$4·85?

$$\frac{1 \times 112 \times 6{\cdot}75}{100 \times 4{\cdot}85} = 1{\cdot}559 = £1\ 11s.\ 2d.$$

Now take the *unit* of value \$1 as the price per 100 lbs.

$$\frac{1 \times 1 \times 112}{100 \times 4{\cdot}85} = {\cdot}231. \quad \$6{\cdot}75 \times {\cdot}231 = 1{\cdot}559 = £1\ 11s.\ 2d.$$

Then it follows that whatever may be the price per 100 lbs. in dollars, that price multiplied by ·231 will show the price of 112 lbs. in £ sterling.

The standard weight and fineness respectively of the Indian Rupee is 180 grains and 916·6 millesimal fineness, standard silver is 925 fine, what is the value of the Rupee in pence, standard silver being worth 50 pence per oz.?

$$\frac{50 \times 180 \times 916{\cdot}6}{480 \times 925} = 18{\cdot}58 \text{ pence. } \textit{Ans.}$$

Now find the value of the Rupee, silver being 1d. per oz.

$$\frac{1 \times 180 \times 916{\cdot}6}{480 \times 925} = {\cdot}3716. \quad 50d. \times {\cdot}3716 = 18{\cdot}58 \text{ pence.}$$

The market price of silver in pence × ·3716 will show the market value of the Rupee in pence.

If wine in France is worth 85 francs per hectolitre, what is the cost of 1 imp. gallon in £ stg., 20 fr. being = 15s. 10¼d.?

$$\frac{{\cdot}7927083 \times 85 \times 1}{20 \times 22{\cdot}00965} = {\cdot}153 = £0\ 3s.\ 0\tfrac{3}{4}d.$$

Now find the cost of 1 gallon at 1 franc per hectolitre.

$$\frac{{\cdot}7927083 \times 1 \times 1}{20 \times 22{\cdot}00965} = {\cdot}0018. \quad 85 \times {\cdot}0018 = {\cdot}153 = £0\ 3s.\ 0\tfrac{3}{4}d.$$

Then it follows that the price per hectolitre in francs × ·0018 = the price per gallon in £ sterling.

The price per hectolitre in francs ÷ 22 = the price per imperial gallon in francs.

An *aliquot* part of a number is a *measure* of that number, or such a part as will exactly divide it; thus, $12\frac{1}{2}$ is an aliquot part of 100 because it is contained 8 times *exactly* in 100; 1s. 8d. is an aliquot part of £1 because it is contained in £1 12 times. The aliquot parts of £1 are 10s., 6s. 8d., 5s., 4s., 3s. 4d., 2s. 6d., 2s., 1s. 8d., 1s. 4d., 1s. 3d., 1s., 10d., 8d., 6d., 4d., 3d., 2d., &c.

To find the cost when the price of *one* is the aliquot part of a £1. *Regard the given number as pounds sterling, and multiply by the aliquot part.*

The aliquot parts of a florin are 1s., 8d., 6d., 4d., 3d., $1\frac{1}{2}$d., &c. To find the cost when *one* is the aliquot part of 2s. *Regard the given number of articles as* £1 *sterling, divide by* 10, *and multiply by the aliquot part.*

Find the cost of 738 articles at 4d. each, $\frac{73 \cdot 8}{6}$ £12·3 = £12 6s. 0d.

The aliquot parts of one shilling are 6d., 4d., 3d., 2d., $1\frac{1}{2}$d., $\frac{3}{4}$d. To find the cost when the price of *one* is the aliquot part of a shilling. *Regard the given number as shillings, and* $\times$ *the aliquot part.*

The aliquot parts of a ton are 10cwts., 5cwts., 4cwts., $2\frac{1}{2}$cwts., 2cwts., 1cwt., $\frac{1}{2}$cwt., &c. The aliquot parts of a cwt. are 56lbs., 28lbs., 14lbs., 7lbs., 4lbs., $3\frac{1}{2}$lbs., &c.

To multiply by the aliquot parts of 100 or 1000. *Reduce mixed numbers to decimals, multiply by* 100 *or* 1000, *and take the aliquot part.*

To divide by the aliquot part of 100 or 1000. *Remove the decimal point two or three places left, and multiply by the denominator of the aliquot part.*

$6\frac{1}{4}$ is $\frac{1}{16}$ of 100	$62\frac{1}{2}$ is $\frac{5}{8}$ of 100	250 is $\frac{1}{4}$ of 1000
$8\frac{1}{3}$ is $\frac{1}{12}$ of 100	$66\frac{2}{3}$ is $\frac{2}{3}$ of 100	$312\frac{1}{2}$ is $\frac{5}{16}$ of 1000
$12\frac{1}{2}$ is $\frac{1}{8}$ of 100	75 is $\frac{3}{4}$ of 100	$333\frac{1}{3}$ is $\frac{1}{3}$ of 1000
$16\frac{2}{3}$ is $\frac{1}{6}$ of 100	$83\frac{1}{3}$ is $\frac{5}{6}$ of 100	375 is $\frac{3}{8}$ of 1000
$18\frac{3}{4}$ is $\frac{3}{16}$ of 100	$87\frac{1}{2}$ is $\frac{7}{8}$ of 100	500 is $\frac{1}{2}$ of 1000
25 is $\frac{1}{4}$ of 100	$62\frac{1}{2}$ is $\frac{1}{16}$ of 1000	625 is $\frac{5}{8}$ of 1000
$31\frac{1}{4}$ is $\frac{5}{16}$ of 100	$83\frac{1}{3}$ is $\frac{1}{12}$ of 1000	$666\frac{2}{3}$ is $\frac{2}{3}$ of 1000
$33\frac{1}{3}$ is $\frac{1}{3}$ of 100	125 is $\frac{1}{8}$ of 1000	750 is $\frac{3}{4}$ of 1000
$37\frac{1}{2}$ is $\frac{3}{8}$ of 100	$166\frac{2}{3}$ is $\frac{1}{6}$ of 1000	$833\frac{1}{3}$ is $\frac{5}{6}$ of 1000
50 is $\frac{1}{2}$ of 100	$187\frac{1}{2}$ is $\frac{3}{16}$ of 1000	875 is $\frac{7}{8}$ of 1000

DECIMALS.

The system of Decimal fractions is so pre-eminently simple, that when it is generally understood it will entirely displace the clumsy system of common fractions. In harmony with our system of notation, it is a fraction always having some power of ten for a denominator: thus $.1 = \frac{1}{10}$, $.03 = \frac{3}{100}$, $.007 = \frac{7}{1000}$, $47.8 = 47\frac{8}{10}$, &c., &c.

Where common fractions occur the calculation may be often simplified by reducing them to decimals.

To reduce a common fraction to a decimal.

RULE. *Divide the numerator by the denominator.*

$\frac{1}{2} = .5$ $\frac{1}{4} = .25$ $\frac{1}{8} = .125$ $\frac{1}{16} = .0625.$
$\frac{3}{4} = .75$ $\frac{1}{3} = .33^{33}$ $\frac{2}{3} = .66^{66}$ $\frac{1}{5} = .2$ $\frac{2}{5} = .4$
$\frac{4}{5} = .8$ $\frac{3}{5} = .6$ $\frac{1}{6} = .16^{66}$ $\frac{1}{9} = .11^{11}$ $\frac{1}{12} = .083^{33}$

Decimals of £1 sterling. See page 59.

2s. $= \frac{1}{10} = \cdot 1$, 1s. $= \frac{1}{20} = \cdot 05$, 5s. $= \frac{5}{20} = \cdot 25$, 6d. $= \frac{1}{40} = \cdot 025$
3d. $= \frac{1}{80} = \cdot 0125$, 1d. $= \frac{1}{240} = \cdot 0041\dot{6}$, $\frac{1}{4}$d. $= \frac{1}{960} = \cdot 001041\dot{6}$,

ADDITION OF DECIMALS

Is performed in the same manner as in whole numbers; care being taken to place the numbers to be added so that the decimal points are in a perpendicular line, and place the decimal point in the *sum* under those in the numbers added.

SUBTRACTION OF DECIMALS

Is performed in the same manner as in whole numbers, care being taken to place the decimal point in the subtrahend under that in the minuend, and the decimal point in the remainder under those in the numbers employed.

MULTIPLICATION OF DECIMALS.

Rule. Multiply as in whole numbers, and point off as many places to the left for decimals as there are decimal places in both factors.

1. Multiply .5 by .5. Ans. .25.
2. Multiply 1.75 by .3. Ans. .525.
3. Multiply 27.46 by .4 Ans. 10.984

When there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers. $.3 \times .3 = .09$. $.29 \times .004 = .00116$.

To multiply by .1 remove the decimal point *one* place to the *left*, by .01 *two* places, by .001 *three* places, by 10 *one* place to the right, by 100 *two* places, by 1000 *three* places, &c., &c.

Note. In practical business the answer to *three* decimal places is sufficiently exact, the *third* decimal only counting for mills, the drudgery of finding, and writing the figures for decimals of no value, may be avoided by reversing the order of the multiplier and writing the first figure of the reversed multiplier under the third decimal figure in the multiplicand, begin each line of the partial products, with the product of the multiplying figure and the figure directly above it, adding the carrying figure, if any, from the immediate right hand figure.

What is the par value in American gold coin of £11 ,, 4 ,, 3, Sterling?

£11.2125	11.2125
4.8665	56 684
560625	44 850
672750	8 970
672750	673
897000	67
448500	5
$54.56563125	$54.565

This example illustrates the difference of the two methods.

DIVISION OF DECIMALS.

The division of decimals is performed in the same manner as in whole numbers, care being taken to point off the decimal places in the quotient.

Rule. Divide as in whole numbers, and point off in the quotient as many places to the left for decimals as the decimal places in the dividend exceed those in the divisor.

Divide .244 by .4. Ans. .61.

Divide .255 by .05. Ans. 5.1.

Divide 776 by 4.2 Ans. 184.76+

Divide 271 by 3.1416 Ans. 86.26+

Divide 3.1416 by .7854 Ans. 4

Divide 500 by 4.8665 Ans. 102.743+

Find the *Par* value in Pounds Sterling of $54.5656 U. S. Gold Coin. See pages 57, 59 and 100. Ans. £11,,4,,3.

The learner can supply additional examples at discretion, bearing in mind the following: The *dividend* must always contain, at least, as many decimal places as the *divisor*. When the number of figures in the quotient is less than the excess of the decimal places in the *dividend* over those in the *divisor*, the deficiency must be supplied by prefixing ciphers. When there is a remainder after dividing the dividend, annex ciphers, and continue the division; the ciphers annexed are decimals to the dividend.

To divide by any number expressed by 1 and any number of ciphers, remove the decimal point as many places to the left as there are ciphers in the divisor.

$74864 \div 1000 = 74.864$

To Divide by adding the Difference of 10, 100, 1000, etc. when the Divisor is something less than any power of ten.

Rule. To the left and in a line with the Dividend write the Divisor; under; write the Difference; find the first figure in the Quotient and with it Multiply the Difference, add the Product to the part of the number divided, write down the sum, with the next figure in the dividend annexed, point off the left hand figure, and so proceed to the end.

Divide 99847632 by 97 Ans. $1029357\frac{3}{97}$.

```
97)99847632(1029357        97)99847632(1029357
 3)10,284                     97
    2,907                     284 common method
     9,346                    194
      3,553                    907
       5,682                   873
        7,03                    346
                                291
                                 553
                                 485
                                  682
                                  679
                                    3
```

The *Difference* of 97 and 100=3. The left hand figures pointed off are the same as the Quotient, and serve to check and prove the answer.

The labor of finding the answer to valueless decimals may be saved by cutting off a figure from the right hand of the divisor, as each new figure in the quotient is found, carrying what would have been obtained by the multiplication of the figure cut off, 1 if the multiplication produces more than 5 and less than 15, 2 if more than 15 and less than 25, etc.

```
73.412)648.7654386(8.8373       73.412)648.7654386(8.8373
       587.296                         587.296
        61469|4                         61469
        58729|6                         58730
         2739|83                         2739
         2202|36                         2202
          537|478                         537
          513|884                         514
           23|5946                         23
           22|0236                         22
            1|5710                          1
```

PROPORTION.

Proportion is the equality of ratios; also the comparison of ratios.

Ratio is the relation which one quantity bears to another of the same kind, with reference to the number of times that the one is contained in the other; the Quotient is the Ratio.

Thus, the ratio of 7 to 21 is 3, because 7 is contained 3 times in 21, or 21 is 3 times seven. The same result is obtained if we divide 7 by 21, for we then find $\frac{7}{21} = \frac{1}{3}$, which means that 7 is $\frac{1}{3}$ of 21, and this expresses the very same relation as before, to say that 7 is $\frac{1}{3}$ of 21 is precisely the same as to say that 21 is 3 times 7. The ratio of 9 to 27 is 3, but we have seen that the ratio of 7 to 21 is also 3, therefore, the ratios of 7 to 21 and 9 to 27 are the same, $21 \div 7 = 27 \div 9$, and these quantities are thefore called proportionals.

In any proportion, as $7 : 21 :: 9 : 27$ the product of the middle numbers, 21 and 9, is equal to the product of the extremes, 7 and 27; hence the *Rule*, that when the fourth proportional is unknown,

Multiply the second and third terms, and divide the product by the first.

Example. If 7 sheep cost 21 dollars, what will 9 cost at the same rate? 27 dollars, Ans.

$$\overline{21 \times 9} \div 7 = 27, \text{ or, } \frac{21 \times 9}{7} = 27$$

Note. The first and third terms are of the same kind; the second term is of the same kind as the required Term.

All Arithmetical business problems may be solved by Proportion: in fact, an exact knowledge of the principles of *Proportion* and skill in *Cancellation* are the essential qualifications of a *Good Calculator*.

CANCELLATION

is the process of abridging operations in division by canceling equal factors from both divisor and dividend.

Note.—Be careful to write all the terms in the denomination, or its largest possible fractions, of the required denomination; that is, feet or fractions of a foot; pounds, or fractions of a pound, etc.

CANCELING IN CALCULATION. Whenever it is required to multiply two or more numbers together, and divide by a third, the first step is to state the problem in its most manageable form; this can only be done by the use of the arithmetical signs.

The statement $\dfrac{28 \times 12}{14}$

is to be read, 28 multiplied by 12 is to be divided by 14.

Stating the problem as above we see at a glance if the divisor is contained, and how many times, in either of the multipliers.

In the foregoing example the divisor, 14, is contained twice in the multiplier, 28; then cancel the 14 and substitute 2 for the 28, and say, twice 12 is 24 the answer.

Process, $\dfrac{28 \times 12}{14} \qquad \dfrac{\overset{2}{\cancel{28}} \times 12}{\cancel{14}} = 24.$

EXAMPLE. If 9 turkeys cost \$18, what will be the cost of 27?

$$\frac{18 \times 27}{9} \qquad \frac{18 \times \overset{3}{\cancel{27}}}{\cancel{9}} = \$54, \text{ Answer.}$$

If the divisor is not contained evenly in either of the multipliers, there may be a common divisor for the divisor itself and one of the multipliers; if so, the common divisor may be used in canceling, thus:

$$\frac{63 \times 8}{27} \qquad \frac{\overset{7}{\cancel{63}} \times 8}{\underset{3}{\cancel{27}}} = 18\tfrac{2}{3}, \text{ Ans.}$$

A glance shows that 9 is the common divisor for 63 and 27.

When a common divisor has been used to change the expression of the divisor and one of the multipliers, the new divisor may be cancelled when it is contained an even number of times in the other multiplier.

EXAMPLE

$$\frac{63 \times 8}{36} \qquad \frac{\overset{7}{\cancel{63}} \times \overset{2}{\cancel{8}}}{\cancel{36}\ \ \cancel{4}} = 14.$$

Process 36 and 63 divided by 9, the common divisor, becomes 4 and 7 respectively, the 4 into 8, 2 times, cancel 4 and 8, and twice 7 is 14, the answer.

Summary of the rapid process for *canceling.*

1. Draw a horizontal line; above the line write dividends only; below the line write divisors only.

2. If there are ciphers above and below the line, erase an equal number on either side; 1 standing alone may be disregarded.

3. If the *same* number stands above and below the line, erase them *both.*

4. If any number on either side of the line will divide any number on the other side of the line without a remainder, divide, and erase the two numbers, retaining the quotient figure on the side of the larger number.

5. If any two numbers on either side have a common divisor, divide them by that number, and retain the quotients only.

6. Multiply all the numbers above the line for a dividend, and those below the line for a divisor; divide, and the quotient is the answer.

7. Write all the terms of the same kind in units, or the largest fractions possible, of the same denomination; *i.e.*, feet, or fractions of a foot; yards, or fractions of a yard, &c., &c.

EXAMPLE. If 7 inches of velvet cloth cost $2\frac{1}{2}$ dollars, what will be the cost of 7 yards? $90. Ans.

Process,

$$\frac{5\times7\times36}{2\times1\times\ 7}\qquad\frac{5\times\cancel{7}\times\overset{18}{\cancel{36}}}{\cancel{2}\times1\times\cancel{7}}=90.$$

NOTE. $2\frac{1}{2}$ dollars $=\frac{5}{2}$, 7 yards $=\frac{7}{1}$, 7 inches $=\frac{7}{36}$ of a yard, $\frac{7}{36}$ inverted is $\frac{36}{7}$.

If an upright line is used put dividends on the right, and divisors on the left. In stating a question put the term of the same kind as the required term first, at the top, on the right of the line; then the other terms in pairs of the same kind; if the conditions tend to increase, put the larger term on the right of the line; if otherwise, on the left.

EXAMPLE: If 5 compositors, in 16 days of 14 hours long, can compose 20 sheets of 24 pages in each sheet, 50 lines in a page, and 40 letters in a line, in how many days of 7 hours long may 10 compositors compose a volume containing 40 sheets, 16 pages in a sheet, 60 lines in a page, and 50 letters in a line, 1 of the second set of compositors being equal to 2 of the first? Ans. 16 days.

Days		16	required term.
Compositors .	10	5	less time with 10 than 5 men.
Hours	7	14	more days with 7 than 14 hours a day.
Sheets	20	40	more time to set 40 than 20 sheets.
Pages	24	16	less time to set 16 than 24 pages.
Lines	50	60	more time to set 60 than 50 lines.
Letters	40	50	more time to set 50 than 40 letters.
Ratio	2	1	

NOTE. Excepting the upper term 16, the numbers on one side exactly balance the numbers on the other, and may all be canceled.

This method acts like a pair of scales, we use known to find the value of unknown quantities; the arrangement of the terms is so very plain and natural as to be easily apprehended; by its use the most complex problems are simplified, and all business calculations made with very few figures, and very little mental effort; it is accurate, and free from the risk of error.

PERCENTAGE.

PER CENTAGE is the term applied to operations in which 100 is the basis of calculation.

The total result obtained by taking a specified number of hundredths of any number is called the *Per Centage.*

PER CENT., from *per centum*, by the hundred; one hundredth of any number=1 per cent.; two hundredths=2 per cent.; five hundredths =5 per cent., &c.

THE RATE per cent. is the number of hundredths taken of each hundred.

Interest, Discount, Broker's fees, etc., etc., are calculated on the basis of an agreed price per cent.

The number of which any per cent. is taken is called the *Base*, or the *Principal.*

The *Base* × the *Rate* = the *Per Centage.*
The *Base* plus the *Per Centage*=the *Amount.*
The *Base*×1, plus the *Rate*=the *Amount.*
The *Per Centage*÷the *Base*=the *Rate.*
The *Amount*÷the *Base*=1, plus the *Rate.*
The *Per Centage*÷the *Rate*=the *Base.*
The *Amount*÷1, plus the *Rate*=the *Base,*
The *Amount*—the *Base*=the *Per Centage.*

RECKONINGS MADE AT A GIVEN RATE PER CENT.

One per cent. means 1 for each 100, and is shown by writing the principal in figures, and then placing a decimal point between the second and third figures from the right, thus: 1 per cent. of 100 = 1·00 = 1. 1 per cent. of 750 = 7·50 = $7\frac{1}{2}$. 1 % of 375 = 3·75 = $3\frac{3}{4}$. 1 % of 780 = 7·8 = $7\frac{8}{10}$.

To find the percentage on any quantity at any Rate per cent. Remove the decimal point two places to the left and multiply by the Rate. 4 % of 750=7·5×4=30·0=30.

2 % of £487 10s. 0d.=4·875×2=9·75=£9 15s. 0d.

$3\frac{1}{4}$ % of £275 7s. 6d.=2·75375×$3\frac{1}{4}$=8·95=£8 19s. 0d.

See pages 55, 59, &c.

Sold a horse for £40, lost 20 ℔ ct. What did it cost?

$1-.20=\frac{80}{100}=\frac{8}{10}$ $8 \mid \frac{10}{40}=50$ *pounds.*

The population of a village increased from 900 to 1200, at what rate per cent. did it increase?

$\frac{300}{9}=33\frac{1}{3}$ *per cent.* Or, $\frac{1200}{900}=1.33\frac{1}{3}$.

The sales of a firm fell off from £12000 to £9000, what was the rate per cent. of decline?

$\frac{300}{12}=25$ *per cent.*

Bought a horse for £80, sold it for £105. What per cent. profit? $\frac{25}{80}=31\frac{1}{4}$ *per cent.* Or, $\frac{105}{80}=1.31\frac{1}{4}$.

Bought a piano for £300, sold it for £250. What per cent. loss? $\frac{50}{300}=16\frac{2}{3}$ *per cent.*

Bought a horse for £40. What must it be sold for to gain 20 per cent.?

$\overline{40\times 20}+40=48$, or £$40\times 1.2=48$ Pounds.

How many of 500 sheep will be left, if 20 per cent. of them are sold? $20\%=\frac{1}{5}$, $500-\frac{1}{5}$ of $500=400$, or $500\times .20=100$. $500-100=400$ *sheep.*

What per cent. of 300 is 75? $75\div 300=25$ per cent.

Of what number is 48, 8 per cent.? $48\div .08=600$.

Sold a horse for £60, made 25 per cent., what did it cost? $1+25=\frac{125}{100}=\frac{5}{4}$. $5 \mid \frac{4}{60}=$ £48.

How many dollars will earn 1 cent a day at 9 per cent. per annum?

$360\div 9=40$. *Ans.* \$40.

Find the commission at 2 per cent., and the net proceeds on £147 15s.

£147·75 × ·02 = 2·955 = £2 19s. 1d. = the commission.

£147 15s.—£2 19s. 1d. = £144 15s. 11d. = net proceeds.

INTEREST.

INTEREST is the price or sum charged for the use of money. The sum of money bearing interest is called the *Principal;* Simple interest arises from the use of the *Principal* only. The *Rate* per cent. is the *number* of units charged for the use of each hundred units.

The *Common Method* of reckoning interest in the United States is based on a year of 360 days; in England and U. S. Courts interest is reckoned on the basis of a year of 365 days; when using the *common* method, count thirty days *only* for each entire month and the difference, if any, will be unimportant. *(See Note below).*

Find the interest by both methods on any example from March 3rd to July 27th; $\frac{144}{360}$ exactly equals $\frac{146}{365}$.

COMMON METHOD, GENERAL RULE TO RECKON INTEREST.

$$\frac{\textit{The Principal} \times \textit{the number of days} \times \textit{the Rate}}{} \div 360 \times 100 = \textit{the Interest.}$$

ENGLISH METHOD, GENERAL RULE TO RECKON INTEREST.

$$\frac{\textit{The Principal} \times \textit{the } \text{EXACT} \textit{ number of days} \times \textit{the Rate}}{} \div 365 \times 100 = \textit{the interest.}$$

TO FIND THE DIFFERENCE OF TIME BETWEEN TWO DATES.

Rule. Subtract the earlier from the later date.

EXAMPLE. For what time must Interest be charged on a debt due April 12th, 1882 and settled June 24th, 1883?

$$\text{Process, } \begin{array}{rrr} 83 & 6 & 24 \\ \underline{82} & \underline{4} & \underline{12} \\ 1 & 2 & 12 \end{array} \quad \text{Ans. 1 yr. 2 mos. 12 days.}$$

TO FIND THE NUMBER OF DAYS BETWEEN TWO DATES.

Common Method. Multiply the number of entire months by *three*; call the Product tens and add the extra days; for the *English Method* add *one* day for each month of 31 days; when February occurs, deduct two days for the *Common* year, one day for *Leap* year.

TO RECKON INTEREST ON £ STERLING AT 5 PER CENT. One tenth of the principal is the interest for 2 years, at 5 per cent.

Multiply ·1 *of the principal by half the given number of years.*

Find the interest on £240 10s. for 8yrs. 8mos. at 5 per cent.

yrs. mos. yrs.
8 : 8 ÷ 2 = $4\frac{1}{3}$. £24·05 × $4\frac{1}{3}$ = 104·216 = £104 4s. 4d.

NOTE. If the *exact* number of days in each month is taken for a multiplier, and 360 used for a divisor, the difference or excess will be $\frac{1}{73}$; about $1\frac{1}{3}$ cents to be taken off each 100, or *one penny* off each *six shillings* of interest.

Any number of Pounds Sterling regarded as *shillings* is equal to the *Interest* for one year at 5 per cent. per annum.

Any number of £ sterling, regarded as *pence*, is the interest for *one month*, at 5 per cent. per annum: ·1 of the principal in £ sterling, regarded as *pence*, is the interest for *three days*; hence the following Rules:—

Multiply the principal by the given number of months, and parts of a month; or multiply ·1 *of the principal by one-third the given number of days; the answer will be in pence. Or multiply* ·1 *of the principal by half the number of months, and divide by* 12, *the answer will be in £ sterling.*

Find the interest on £428 from July 27th, 1882, to March 3rd, 1883. £12 16s. $9\frac{1}{2}$d. Ans.

$$\left.\begin{array}{rrr} \text{yrs.} & \text{mo.} & \text{dys.} \\ 83 & 3 & 3 \\ 82 & 7 & 27 \\ \hline & 7 & 6 \end{array}\right\} \begin{array}{l} =216\text{dys.} \\ 7\cdot2\text{mos.} \end{array} \quad \frac{42\cdot8\times216}{3\times12\times20} \text{ or } \frac{42\cdot8\times7\cdot2}{2\times12} \text{ or } \begin{array}{rrrr} \text{d.} & £ & \text{s.} & \text{d.} \\ 428= & 1 & 15 & 8 \\ & & & 7\frac{1}{5} \\ \hline & £12 & 16 & 9\frac{3}{5} \end{array}$$

See note page 46.

The Int. on £510 10s. for 1 mo. at 5%=d. $510\frac{1}{2}$=£2 2s. $6\frac{1}{2}$d.

One-tenth of any number of £ regarded as pence multiplied by *twice* the given Rate is the Int. for 1 month: thus the Int. on £510 10s. for 1 month at $2\frac{1}{2}$% = d. 51·05 × 5 = d. 255·25 = £1 1s. $3\frac{1}{4}$d.

Interest at 5 per cent. ×·2=1 per cent.; × ·4 = 2 per cent.; × $\frac{1}{2}$ = $2\frac{1}{2}$ per cent.; ×·6 = 3 per cent.; — $\frac{1}{3}$ = $3\frac{1}{3}$ per cent.; × ·7= $3\frac{1}{2}$ per cent.; — $\frac{1}{4}$=$3\frac{3}{4}$ per cent.; × ·8 = 4 per cent.; ×·9 = $4\frac{1}{2}$ per cent.; + ·1 = $5\frac{1}{2}$ per cent.; + ·2 = 6 per cent.; + ·3 = $6\frac{1}{2}$ per cent.; + ·4 = 7 per cent., &c., &c., &c.

TO RECKON INTEREST ON £ STERLING AT 6 PER CENT.

One hundredth of the principal is the interest for *two months* at 6 per cent.; ·001 of the principal is the interest for *six days;* ·1 of the principal in £ is the interest in *shillings* for *one month*; ·01 of the principal in £ is the interest in shillings for *three* days; hence the following Rules:

Multiply ·01 *of the principal by half the given number of months; or multiply* ·001 *of the principal by* $\frac{1}{6}$, *the number of days; to have the answer in shillings multiply* ·1 *of the principal by the number of months; or multiply* ·01 *of the principal by* $\frac{1}{3}$, *the number of days.*

Find the interest on £428 10s., from March 3rd to July 27th.

See note on pages 46, also 59.

$$\left.\begin{array}{rr} \text{m.} & \text{dys.} \\ 7 & 27 \\ 3 & 3 \\ \hline 4 : & 24 \end{array}\right\} \begin{array}{l} =144\text{dys.} \\ 4\cdot8\text{mos.} \end{array} \quad \frac{4\cdot285\times4\cdot8}{2} \text{ or } \frac{\cdot4285\times144}{6} = 10\cdot284 = \begin{array}{rrr} £ & \text{s.} & \text{d.} \\ 10 & 5 & 8 \end{array}$$

TO RECKON INTEREST AT 1 PER CENT PER MONTH.

Rule No. 1. *Multiply the Principal by $\frac{1}{3}$ the given number of days and remove the decimal point three places to the left.*

Find the interest on \$143 for 33 days at 1% per month.

$$\overline{\$143\times11}\div1000=\$1.573 \quad \text{Ans.}$$

Find the interest on \$428.50 from March 5th to July 29th.

Mos.	Days.
7	29
3	5
4	24 =144 days.

$$\overline{\$428.50\times48}\div1000=\$20.568 \quad \text{Ans.}$$

Rule No. 2. *Multiply the Principal by the time in months and fractions of a month and remove the decimal point two places to the left.*

Find the interest on the above examples by this Rule.

$$\overline{\$143\times1.1}\div100=\$1.573 \quad \text{Ans.}$$

$$\overline{\$428.50\times4.8}\div100=\$20.568. \quad \text{Ans.}$$

Interest at 1% per month is equal to 12% per annum; interest at 12% per annum $\div4=3\%$; $\div3=4\%$; $\times\frac{5}{12}=5\%$; $\div2=6\%$, $\times\frac{7}{12}=7\%$; $\times\frac{5}{8}=7\frac{1}{2}\%$; $\times\frac{2}{3}=8\%$; $\times\frac{3}{4}=9\%$; etc.

TO RECKON INTEREST BY CANCELLATION.

1st. On the right of an upright line write the Principal, the time,—in days—and the Rate per cent.

2nd. On the left the number of days, or its factors, in the year, and remove the decimal point two places to the left.

Find the interest on \$428.50 at 5% per annum of 360 days, from March 3rd to July 27th. Ans. \$8.57

Mos.	Days.
7	27
3	3
4	24=144 days

$$\begin{array}{rr|l} \cancel{100} & \cancel{6} & 428.5 \\ & \cancel{6} & \cancel{144}\ 2 \\ \cancel{2} & \cancel{10} & \cancel{5} \end{array}\Bigg\}=\$8.57$$

Find the interest on \$99 at 4% for 72 days.

$$\frac{99\times72\times4}{36\times10\times100}=.792 \quad \text{Ans.}$$

Find the interest on £428,,10 Stg. at 5% per annum of 365 days from March 3rd to July 27th. Ans. £8,,11,,5.

Mos.	Days.
7	27
3	3
4	24+2 days=146 days.

$$\begin{array}{r|l} \cancel{100} & 428.5 \\ \cancel{73} & \cancel{146}\ 2 \\ \cancel{5} & \cancel{5} \end{array}\Bigg\}=8.57=£8,,11,,5.$$

See note on pages 46 and 59.

TO RECKON INTEREST AT ONE PER CENT. PER ANNUM.

COMMON METHOD.

RULE. *To find the interest for one year, divide the Principal by* 100; *to find the interest for* 36 *days remove the decimal point three places to the left; for any other time or Rate, increase or diminish in the manner shown in the following examples.*

Find the interest on $1000 for 11 years, 1 month and 6 days, at 1 per cent. per annum. Ans. $111.00

		YR.	MO.	DA.	
$ 10.00	int. for	1	0	0	at 1% = the Principal ÷ 100
100.00	"	10	0	0	= first line × 10
1.00	"		1	6	= " " × .1
111.00	"	11	1	6	at 1% per annum.

Find the interest on £124,,10,, Stg., from March 6th, 1882 to May 18th, 1883, at 7% per annum. Ans. £10,,9,,2.

		YR.	MO.	DA.	
1.245 =	int. for	1	0	0	at one per cent.
.2075 =	"		2	0	1st line × ⅙, 2 mos. = ⅙ of 1 year
.0415 =	"			12	2nd line × .2 12 days = .2 of 60 days
1.494	"	1	2	12	at one per cent.

10.458 = 4th line × 7 = Interest at 7% = £10,,9,,2

The interest is found on all sums at 1 per cent. a month by removing the decimal point to the left, 3 places for 3 days, and 2 places for 30 days.

Find the interest on £143 for 1 mo. 3 da. at 1 per cent per month. Ans. £1,,11,,5½.

£1.43	int. for	1 mo.
.143	"	3 days 1st line × .1
£1.573		

When the given rate is not a convenient part of *five* or *six* per cent., find the interest for the given time at *one* per cent., and multiply by the given rate. Or multiply by twice the rate and remove the point one place to the left; the answer will be in shillings, 3½ % on £100 = 10·0 × 7 = 70 = £3 10s.

NOTE 1. The Decimal expression of values has the same significance whether the examples are stated in terms of the Pound Sterling, the Dollar, or any other Standard Coin. (See page 59.)

NOTE 2. Only a sufficient number of examples to clearly illustrate the working of the several rules are presented; the Teacher or the Student may furnish additional examples for exercises to any extent.

NOTE 3. The answer to *three* decimal places is sufficiently exact; the time being less than one year, use only two decimal places in the principal

For Reckoning Interest at 5 *per cent per year of* 365 *days.*

$$\overline{1\times 1} \div \overline{73\times 100} = \overline{1\times 1\times 5} \div \overline{365\times 100}$$

RULE. *Multiply the Principal by the given number of days, remove the decimal point two places to the left and divide by* 73.

Find the interest on £100 Stg. for 365 days at 5% a year.

$$\frac{£100\times 365}{100\times 73}=£5 \quad \text{Ans.}$$

RULE No. 2. *To Reckon Interest at* 5 *per cent per annum.*

$\overline{(1\times 365+\frac{1}{3})+\frac{1}{10} \text{ and } \frac{1}{100} \text{ of } \frac{1}{3}}\div 10{,}000=.05$ nearly; the excess is exactly $\frac{1}{100000}$, hence the following.

RULE. *Multiply the Principal by the given number of days, Divide the Product by* 3, *and to it add the Quotient plus* .1, *plus* .01 *and remove the decimal point four places to the left.*

Find the Interest on £100 for 365 days at 5% per annum.

3)36500=100×365
121666=the quotient.
12166=.1 Do.
1216=.01 Do.
£5.0004=The Answer.

To Reckon Interest on the basis of a year of 365 *days.*

$$\overline{1\times 1\tfrac{3}{20}}\div 1000=\overline{1\times 42}\div\overline{365\times 100} \text{ nearly.}$$

The deficiency is about .0006 or $\frac{1}{1680}$; six cents to be added to each \$100; one penny to each £7 of interest.

RULE. *Multiply the Principal by* $1\frac{3}{20}$, *remove the point three places to the left and the interest will be shown for*

42 days at	1 per cent,	12 days at	$3\frac{1}{2}$ per cent,
21 "	2 "	7 "	6 "
14 "	3 "	6 "	7 "

divide this interest by the number of days opposite the given rate and the Quotient is the interest for one day, multiply by the given number of days and add .0006 *to the product.*

NOTE. To Multiply by $1\frac{3}{20}$ add $\frac{1}{10}$ and $\frac{1}{2}$ of $\frac{1}{10}$ of any number to itself.

EXAMPLE 1. Find the interest on £100 for 7 days at 6 per cent per annum. Ans. 2s,,$3\frac{3}{4}$d.

100= the Principal,
10=.1=$\frac{2}{20}$ of the Principal,
5= $\frac{1}{2}$ of $\frac{1}{10}$=$\frac{1}{20}$ of the Principal,
.115=the interest for 7 days at 6 per cent.

HOWARD'S LIGHTNING RULE FOR RECKONING INTEREST.

Divide 36 by any given Rate and the *Quotient* is the time, in days, in which the Interest on any given sum is equal to .001 of the Principal; in 10 times the Quotient the interest equals .01 of the Principal; in 100 times the interest equals .1; and in 1000 times the Quotient, the interest equals the principal.

Rule. Divide 36 *by the given Rate, Multiply the Principal by the given number of days, divided by the Quotient, and remove the decimal point three places to the left.*

Find the interest on \$1000 for 9 days at 4% per annum.

$$36\div4=9 \qquad \frac{\$1000\times9}{9\times1000}=\$1.000 \text{ Ans.}$$

The interest for 9 days is \$1.000; for 90 days, \$10.00; for 900 days, \$100.00; and for 9000 days, \$1000.00, or the interest is equal to the Principal.

If millions of examples were written together in a column, the same denominations being placed exactly under each other; a straight line drawn three decimal places to the left, from the top of the column to the bottom, would show the interest on each, and every one of these millions of examples for 9 days at 4 per cent. per annum without altering one figure of the Principal; similar lines drawn two places, and one place to the left would show the interest for 90 days, and 900 days; thus doing the work of a long life in a moment of time.

36÷3=12	the divisor at 3%.	36÷9=4	the divisor at 9%
36÷4= 9	" " 4%.	36÷12=3	" " 12%
36÷6= 6	" " 6%.	36÷18=2	" " 18%

Find the interest on £428 Stg. at 9% per annum from July 27th, 1882 to March 3rd, 1883.

YR.	MOS.	DAYS.
83:	3:	3
82:	7:	27
	7:	6=216 days.

$$\frac{428\times\cancel{216}\;54}{\cancel{4}\times1000}=23.112=£23,,2.,3. \quad \text{Ans.}$$

See notes on pages 46 and 59.

COMPOUND INTEREST.

Compound interest is interest on the principal, and also on the interest added to the principal, each time it becomes due.

RULE. Multiply the principal by the rate, setting the product under, and two decimal places to the right of the principal; the sum of principal and interest will be the amount.

Or, find the amount of £1, or $1, for the given time and rate, and multiply by the given principal.

NOTE. To avoid writing decimals of no value, begin at the third decimal adding in the figure carried, if any, from the right hand figures.

Find the amount of £864 10s. 0d. for six years at 8%. Ans. £1371 17s. 0¾d.

NOTE. Persons having frequent occasion to compute compound interest may save time and labor by the use of a table showing the amount of one pound, or one dollar, for a series of years, or other stated periods; the amount of one pound, or one dollar, for the given time and rate, multiplied by the given number of pounds, or dollars, will be the amount sought. (See page 65.)

School Book Method, 184 Figures.

```
           864.5
               8
           69160
          8645
         933.660
               8
         746928
        93366
       1008.3528
               8
       80668224
      10083528
      1089.021024
               8
      8712168192
     1089021024
     1176.14270592
               8
     940914164736
   117614270592
   1270.2341223936
               8
   101618729791488
  1270.2341223936
  £1371.85285.2185088
```

Howard's Method, 74 Figures.

```
  864.5
   69.16
  933.66
   74.693
 1008.353
   80.668
 1089.021
   87.122
 1176.143
   94.091
 1270.234
  101.619
£1371.853
```

Paying *Simple Interest* for fractions of any given single period is usual, but it involves a loss to the Payer, because Simple Interest is more than Compound interest for any portion of any single period. Estimating our National debt at $2,000,000,000 and the average Interest at 5 per cent, the Bondholders gain and the Nation looses $1,890,673 a year by computing the quarterly payments at simple interest.

Every day's true Compound Interest differs, increasing as the day is distant from one.

Every day's true discount differs, decreasing as the day is distant from one,

The product of the amount of £1 for any two periods = the amount for the sum of the two given periods.

The *Square* of the amount of £1 at Compound Interest for any given number of terms = the amount for twice that number of terms.

The *Square Root* of the amount of £1 for any given number of terms = the amount for half that number of terms.

The *Cube* of the amount of £1 for any given number of terms = the amount for three times that number.

The *Cube Root* of the amount of £1 for any given number of terms = the amount for one third that number.

Example. $10,000 invested at 6 per cent. per annum true Compound Interest is to be divided so that each of three sons on becoming 21 years old is to receive an equal sum; A is 17½, B 13⅓ and C 10 years old; find how much each will receive and the present value of each son's share.

1st. Find the present worth of $1 due in 3½, 7⅔ and 11 years, true Compound Interest, then find the *sum* of the results.

2nd. Divide $10,000 by this sum and the *Quotient* will be the amount due when each son comes of age.

3rd. Multiply this result by the present worth of $1, as before found, for each son, and the products will be the present worths required.

1st. $\frac{1}{1.226226}$ = present worth of $1 for 3½ years = $.8155104

$\frac{1}{1.563189}$ = present worth of $1 for 7⅔ years = $.6397176

$\frac{1}{1.898298}$ = present worth of $1 for 11 years = $.5267875

Sum of results, $1.9820155

2nd. $10,000 ÷ 1.9820155 = $5045.37 = sum each son will receive.

3rd. $5045.37 × .8155104 = $4114.55 = present worth of A's share.
" × .6397176 = 3227.61 = " " B's "
" × .5267875 = 2657.84 = " " C's "

Another method. 1st. Find, by Proportion, what sum invested for 3½, and 7⅔ years respectively will equal the amount of $1 for 11 years.

2nd. Multiply each such investment by the $10,000 and divide the product by the sum of the investments; the quotient is the present value of each son's share.

NOTE. By computing simple, instead of true Compound Interest for the fractions of a year, the present values would be $2658.62, $3227.36 and $4114.02, and the amount when of age, $5046.86.

DISCOUNT.

DISCOUNT is a certain per centage deducted from, or allowance made for the payment of a debt or other obligation, before it is due. The PRESENT WORTH of any sum is a sum which if put at interest now at a given rate will amount to the required sum when due. TRUE DISCOUNT is the difference between the *Present Worth* and the amount.

BANK DISCOUNT is simple interest on the Principal for a specified time, with three days added, called *Days of Grace*, a note for 3 months is due 3 months 3 days from date; a note for 90 days is due in 93 days; a possible difference of 2 days.

In reckoning *Bank Discount* the sum on which interest is to be paid, is known, but in reckoning *True Discount* we have to find what sum must be placed at interest so that the sum together with its interest may amount to the given Principal.

To find the present worth of any sum, and the true discount for any time at any rate per cent.

RULE.—*Divide the given sum by the amount of $1 for the given time and rate; the quotient will be the present worth, and the difference will be the discount.*

Find the present worth and the true discount on \$1000 for 1 year at 10 per cent.

$\frac{1000}{1.10}=\$909.09$ present worth, $1000-909.09=\$90.90$ true dis.

Find the Bank Discount on a note for \$1000 for 1 year at 10% per annum. $\$100+.83=\$100.83$. Ans.

$\$100=$interest for 1 year. $.83=$interest for 3 days.

TO RECKON TRUE DISCOUNT. NEW METHOD.

RULE. *Multiply the Principal by the simplest form of the fraction formed by taking the given Rate per cent. for a numerator, and* 100 *plus the percentage for a denominator.*

Find the true discount on £100 for 1 year at 5 %.

$$\frac{5}{105}=\frac{1}{21}\quad £100\times\tfrac{1}{21}=4\cdot7619=£4\ 15s.\ 2\tfrac{3}{4}d.$$

Find the true discount on £149 10s. 6d. for 1 year at 6%.

$$\frac{6}{106}=\frac{3}{53}\quad £149\cdot525\times\frac{3}{53}=8\cdot463=£8\ 9s.\ 3d.$$

EXAMPLE. An agent receives 10% on the net sum paid to his Principal, find his commission on £1000.

$$\frac{10}{110} = \frac{1}{11}. \quad \frac{1000}{11} = 90{\cdot}909 = £90\ 18s.\ 2d.$$

COMMERCIAL DISCOUNT is a given Rate per cent. allowed off a *Debt* or part of a *Debt* for Cash, that is, "ready money;" and is reckoned the same as interest.

A bill of goods is bought, amounting to 960 dollars at a year's credit, the merchant offers to deduct 10% for ready cash, what amount is to be deducted?

$$\overline{\$960 \div 100} \times 10 = \$96.00. \quad \text{Ans.}$$

By discounting the face of bills, a loss may be sustained without suspecting it; this arises from the fact that the discount is not only made on the first cost of the goods, but also on the profits; for instance, if a profit of 30% be made on any article of merchandise, and the 10% be deducted, the gain at first sight would appear to be 20%, but is in reality only 17%. If a profit of 60% be added to the first cost, and then a discount made of 45%, the apparent profit would be 15%; instead of this, an actual loss is made of 12%, as will be seen by the following examples:

Example 1.

Cost of goods,	£100
Add 30% profit,	30
Selling price,	130
Deduct 10% discount,	13
Cash price,	£117
Gain 17%.	

Example 2.

Cost,	£100
Profit 60%,	60
Selling price,	160
Discount 45%,	72
Cash price,	£88
Loss 12%.	

The net amt. of a bill, less 10 per cent. discount, will be shown by multiplying by .9. Example. £100×.9=£90.0

To find the net. amt. less discount at

5 per cent × 9½.	30 per cent × 7.	50 per cent × 5.
15 " " × 8½.	35 " " × 6½.	55 " " × 4½.
20 " " × 8.	40 " " × 6.	60 " " × 4.
25 " " × 7½	45 " " × 5½.	70 " " × 3.

and remove the point 1 place to the left.

PARTNERSHIP.

A *Partnership* or *Firm* is an association of two or more persons for the purpose of transacting business with an agreement to share the profits and losses according to the amount of capital furnished by each, and the time it is employed.

Capital or *Joint Stock* is the amount of money or property belonging to the firm used in the business; the amount due together with the property of all kinds belonging to the firm, is sometimes called the *Assets.*

The *Net Capital* is the excess of assets over liabilities.

The *Liabilities* of a firm are its debts.

To find each partner's share of the profit or loss.

RULE. *Multiply the whole profit or loss by the ratio of the whole Capital to each man's share of the Capital.*

Example, A and B engage in trade, A furnishes \$300 and B \$400, they gain \$91; what is each man's share of the profits?

Capital	\$300, $\frac{300}{700}=\frac{3}{7}$.	Gain,	$\$91\times\frac{3}{7}=\$39$=A's Share.
"	400, $\frac{400}{700}=\frac{4}{7}$.	"	$91\times\frac{4}{7}=52$=B's "
Whole stock,	\$700.	Whole profit,	\$91

Another method: Find the rate per cent. gained or lost, and multiply each person's share of the capital by the rate per cent.

$$\frac{91}{700}=13\%. \qquad \left.\begin{matrix}300\times.13=39\\400\times.13=52\end{matrix}\right\}=\$91.$$

When the respective capital of each partner is invested for unequal periods of time.

RULE. *Multiply each man's capital by the time it is employed, and regard each product as his capital, and the sum of the products as the entire capital.*

Take the above example, A's capital being invested for four months, andB's for three months, and find each man's share of profits.

$\$300\times4=1200$	$\$91\times\frac{12}{24}=\$45.50$=A's share.
$400\times3=1200$	$91\times\frac{12}{24}=45.50$=B's "
Capital 2400	Profits \$91.00

EXCHANGE.

Exchange, in Arithmetic, is a method of finding the value of one denomination of money in the terms of another.

Exchange, in Commerce, is the paying, or receiving any sum in one kind of money for its value in another; when the parties are distant from each other this is done by means of an *Order* or *Draft* called a *Bill of Exchange.* Bills drawn in one Country and made payable in another are called *Foreign Bills;* when drawn and payable in the same country they are called *Inland Bills.*

PAR OF EXCHANGE is the established value of the Standard Coin of one country when expressed in terms of the Standard Coin of another; the value of £1 Stg. in U. S. gold coin is $4.8665. See page 100. Exchange is at *Par* when a Bill in New York, for the payment of £100 Stg in London can be sold for $486.65. Exchange is *in favor* of a place when it can be sold there at or above par; Exchange diverges from Par by the difference in the amount of the indebtedness between one country and another, called the *Balance of Trade.*

Find the value in *currency* of a gold dollar, the market price of currency being 75 cents. Ans. 133⅓ cents.

Process $\frac{100}{75} = \frac{4}{3} = 133\frac{1}{3}$

2. Find the value of currency, the price of gold being 133⅓. Ans. 75 cents.

Process $\frac{100}{133\frac{1}{3}} = \frac{3}{4} = .75$

$500 in gold at 8 per cent. premium will buy how much currency?

$\$500 \times 1.08 = \$540.$

$500 in currency will buy how much gold at 8 per cent. premium?

$500 \div 108 = \$462.96.$

Dollars being worth 49½ pence, to find their value in £ stg.

Regard $\frac{1}{10}$ the number of dollars as £. Multiply by 2 and add as many and half as many pence as there are dollars.

$\$50 = £5.0 \times 2 + 50d. + 25d. = £10\ 6s.\ 3d.$

$\text{Francs} \times \frac{4}{100} = £\text{ stg.} \qquad £ \times \frac{100}{4} = \text{Francs.}$

What is the face value of a bill of Exchange costing £1000. Commission ¾ per cent?

$$£1000 \div 1.0075 = £992.555$$

What is the cost of a bill of Exchange for $1000 Premium ¾ per cent.

$$\$1000 \times 1.00\tfrac{3}{4} = \$1007.50.$$

Find the par value of £473 ,, 5 ,, 9 St'g. in American gold coin.

$$£473.2875 \times 4.8665 = \$2303.25.$$

Note. To avoid encumbering the operation with valueless decimals, reverse the multiplier, and begin each line of the partial products with the product of the multiplying figure and the figure directly above it, adding what otherwise would have been carried.

473.2875
56.684
1893.150
378.630
28.397
2.839
.237
2303.254

The par value of £1 st'g is fixed by act of Congress 1873, at $4.8665.

Pounds Sterling × 4.866563 = the *Par* value of U. S. Dollar.
U. S. Dollars × .2054838 = the *Par* value of Pounds Stg.

BRITISH MONEY.

The current denominations of British money are the Pound Sterling or sovereign and half-sovereign, gold; the florin, shilling, sixpence, and threepence, silver; and the penny, halfpenny, and farthing, bronze; if the farthing was made equal to $\frac{1}{1000}$ of £1 these coins would constitute a Decimal Currency; practically they do so now; reference to the next page will show that the florin, shilling, sixpence, threepence, &c., can be written as decimals of £1, absolutely correct; it also shows that any sum of British money may be written decimally, sufficiently exact for all practical purposes.

Interest, percentage, and some other monetary and commercial reckonings are very much simplified by writing fractional money values as decimals of £1. The fractional coins and the usual fractions of the standard weights and measures have a mutual adaptiveness that greatly facilitates many business calculations; it is therefore often preferable to operate with common fractions.

See pages 31, 35, 100, 118, &c.

NOTE. By carefully observing and practising the following instructions, the converting of shillings, pence and farthings into decimals of a pound, and *vice versa*, will become a purely mental and instantaneous operation.

1. For every two shillings, or florin, write .1, because two shillings is $\frac{1}{10}$ of a pound stg.

2. For every 1 shilling, write .05, because one shilling is $\frac{5}{10}$ of a florin, or $\frac{5}{100}$ of a pound stg.

3. For every ninepence, write .0375, because ninepence is $\frac{375}{10000}$ of a pound stg.

4. For every sixpence, write .025, because sixpence is $\frac{25}{100}$ of a florin or $\frac{25}{1000}$ of a pound stg.

5. For every threepence, write .0125, because threepence is $\frac{1}{8}$ of a florin, or $\frac{125}{10000}$ of a pound stg.

6. For the pence write the farthings therein as thousandths of £1, and add one for each sixpence.

6d.$=$·024$+$1$=$·025. $3\frac{3}{4}$d.$=$·015$+\frac{1}{6}$ of 3·75$=$·015625.
9d.$=$·036$+\frac{1}{6}$ of 9$=$·0375. $4\frac{1}{2}$d.$=$·018$+\frac{1}{6}$ of 4·5$=$·01875.

£		s.		d.		far.
1	=	20	=	240	=	960
.1	=	2	=	24	=	96
.05	=	1	=	12	=	48
.0375	=	$\frac{3}{4}$	=	9	=	36
.025	=	$\frac{1}{2}$	=	6	=	24
.0125	=	$\frac{1}{4}$	=	3	=	12
$.004\frac{1}{6}$	=	$\frac{1}{12}$	=	1	=	4
$.00104\frac{1}{6}$	=	$\frac{1}{48}$	=	$\frac{1}{4}$	=	1

£19,,2	= 19.1	£27,,12,,6	=27.625
19,,3	= 19.15	19,,19,,$2\frac{1}{2}$	=$19.96041\dot{6}$
19,,5	= 19.25	19,,18,,$0\frac{3}{4}$	=19.903125
19,,19	= 19·95	19,,16,,$1\frac{3}{4}$	=$19.80729\dot{1}\dot{6}$
19,,18	= 19.9	24,, 1,,$1\frac{1}{2}$	=24.05625

The answer to three decimal places is sufficiently exact.

Read for each unit in the *first* decimal place 2s.; for each 5 in the *second* place 1s.; for *one-fourth* of the remaining *second* and *third* decimals read Pence: a remainder of 3 call $\frac{1}{2}$d., a remainder less than 3 disregard: thus ·031 = $\frac{31}{4}$ = $7\frac{1}{2}$d., the exact number of farthings = — 4 per cent., thus ·003125 — ·000125 = 3 farthings. The Teacher or the Learner may furnish additional examples for exercises to any extent.

STOCKS AND BONDS.

A Bond is a duly certified document acknowledging the indebtedness, with the limits and conditions of the debt, of a Corporation or a Government.

Government Bonds are sometimes called Consols.

The capital of a company in transferable *Shares*, each of a certain amount, is called Stocks.

The value expressed on the face of any certificate of value, as Stocks, Commercial Paper, etc., is called the *Par Value.* When the market value is greater or less than par value, the Stock is said to be above or below *Par*, or is said to be at a *Premium*, or *Discount*, as the case may be.

To find what Rate per cent. will be gained by money invested in Stocks at any given price.

Rule. *Multiply the rate per cent. dividend by* 100, *and divide by the price paid for each Share.*

Thus the gain per cent. by investing in 4 per cent. Stock at 80 = 400 ÷ 80 = 5 = 5 per cent.

What per cent will be gained by investing in 8 *per cent stock, at* 20 *per cent premium?*

120 | 800=6⅔ *per cent.*

What per cent will be gained by investing in 6 *per cent stock at* 10 *per cent discount.*

100−10=90. 90 | 600=6⅔ *per cent.*

To find at what price stock paying a given rate per cent. dividend can be purchased, so that the money invested shall produce a given rate of interest.

Rule. Divide the rate per unit of dividend by the rate per unit of interest.

What must be paid for stock paying 6 *per cent dividend, in order to realize on the investment* 8 *per cent?*

8 | 600=75.

TAXES.

A Tax is a sum of money assessed on persons or property for the purpose of defraying public expenses.

Real Estate is fixed property, such as houses and lands.

Personal Estate consists of money, cattle, ships, furniture and other movable property.

To find the rate of taxation, the required tax and the value of the taxable property being known:

RULE. *Divide the required tax by the value of the taxable property, the quotient is the rate of taxation.*

Example. The taxable property of a township is valued at $4,835,000, the required tax is $96,700; what is the rate?

$\frac{96{,}700}{4{,}835{,}000}=.02$ Ans. 2%, or 2 cents on each dollar.

The required tax divided by the rate=the valuation.

To find the amount of any person's tax.

RULE. *Multiply the value of the property by the Rate.*

Example. The assessed value of Oscar Wilde's property in Dublin is £48,500; the Rate is $\frac{7}{8}$ of 1%; what is the amount of his Tax?

£48,500$\times$.00$\frac{7}{8}$=424;375=£424,,7,,6 Ans.

DUTIES.

Duties are taxes paid on many kinds of goods imported from abroad and are collected by the Custom house officers; when the duty is a certain per cent. on the value of the goods it is called *Advalorem* duty; when the duty is on a certain quantity, as agreed, a pound, a gallon etc., it is called a *Specific Duty.*

A Tariff is a schedule showing the rates of duties fixed by law on all kinds of imported merchandise.

Gross weight or *value* is the weight or value of the goods before any allowance is made.

Net weight or *value* is the weight or value of the goods after all allowances have been deducted.

INSURANCE.

Insurance is a contract of indemnity against loss or damages.

Fire Insurance is indemnity for loss of property by fire.

Marine Insurance is indemnity for loss of vessels or cargo.

Life Insurance is an agreement to pay a certain sum in case of the death of the insured.

The Insurer or *Underwriter* is the party who takes the risk; the written contract between the two parties is called the Policy.

The Premium is the sum paid for Insurance.

To find the Premium, the sum insured and the Rate being given.

RULE. *Multiply the sum invested by the rate.*

To find what sum must be insured to cover both the Property and Premium, the Rate being given.

RULE. *Divide the value of the Property by* 1 *minus the Rate.*

PROFIT AND LOSS.

To find the gain or loss per cent.

RULE. *Divide the gain or loss by the cost.*

To find the selling price to gain a given per cent.

RULE. *Multiply* 100, *plus the gain per cent. by the cost and divide by* 100.

To mark goods so that a given per cent. may be deducted and yet make a given per cent profit.

RULE. *Divide the real selling price by* 1 *minus the given per cent. to be deducted, the quotient is the marking price.*

Example. Bought hats at \$2.55 each, at what price must they be marked so that 15 per cent. may be deducted, and yet be sold at 20% profit.

\$2.55+20%=\$3.06=the selling price.

3·06÷1−·15 *i.e.* 15 per cent.=\$3·60=the asking price.

To mark goods to gain a given per cent. on the selling price: *Divide the cost by* 1, *minus the required Rate per cent.*

See page 101.

ALLIGATION.

Alligation treats of mixing or compounding two or more ingredients of different values or quantities; the process of finding the *mean* value or quantity of several ingredients, is called *Alligation Medial.*

RULE. *Find the entire cost, or value of the ingredients and divide it by the sum of the simples.*

Alligation Alternate is the process of finding the proportional quantities to be used in any required mixture.

RULE. *Arrange the ingredients in pairs, one of less and the other of greater value than the required value, the difference of one member of a pair and the required value is the required quantity of the other member.*

To prove the answer, multiply each value by its quantity, and divide the sum of the products by the sum of the quantities.

Example. Having four qualities of tea worth 1, 2, 3 and 4 dollars a pound, how much of each must be used to make a mixture worth $2\frac{1}{2}$ dollars a pound.

$$2\tfrac{1}{2}\left\{\begin{array}{l}1\times\frac{1}{2}=\frac{1}{2}\\ 3\times1\frac{1}{2}=4\frac{1}{2}\\ 2\times1\frac{1}{2}=3\\ 4\times\frac{1}{2}=2\\ \hline 4)\ 10=2\frac{1}{2}\end{array}\right. \quad \text{or} \quad 2\tfrac{1}{2}\left\{\begin{array}{l}1\times1\frac{1}{2}=1\frac{1}{2}\\ 4\times1\frac{1}{2}=6\\ 2\times\frac{1}{2}=1\\ 3\times\frac{1}{2}=1\frac{1}{2}\\ \hline 4)\ 10=2\frac{1}{2}\end{array}\right.$$

By the first arrangement we get $\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$ and $\frac{1}{2}$ as the required quantities, in all 4 pounds, costing \$10, an average of $2\frac{1}{2}$ dollars a pound.

Questions in Alligation may have different answers, the preference for any one depends upon the quantity of particular ingredients on hand.

For adjusting the fineness of Gold and Silver, see page 119.

BARTER.

Barter is the exchange of commodities.

RULE. *Divide the given quantity of the given commodity and its price by the constituents of the commodity whose value is required.*

Example. How much tea at 64 cents a pound must be given for 448 pounds of cheese at 20 cents a pound?

$$8\times8=64. \qquad \frac{448\times20}{8\times8}=140 \text{ lbs. Ans.}$$

ANNUITIES.

MODEL OF ANNUITY TABLE, ANNUITY £1, RATE 5 PER CENT.

Years.	Amount.	Present Worth.	Amt. of Annuity.	Pres. Worth of Ann.
1	1.05	.95238095	1	.95238095
2	1.1025	.90702947	2.05	1.85941042
3	1.157625	.86383759	3.1525	2.72324801
4	1.21550625	.82270247	4.310125	3.54595048
5	1.27628156	.78352616	5.52563125	4.32947664

The amount of an annuity of £1=the compound interest on one pound÷the Rate, hence the amount of an annuity of £1 forborne for three years=.157625÷.05=3.1525.

The Present value of an annuity of £1 for any number of terms = the compound interest ÷ the Rate × the amount; thus the Present Worth of an annuity of £1 for three years at 5% $\frac{=.157625}{.05\times1.157625}$ =2.72324801=£2,,14,,5½

The amount of £1 for any given time and rate Multiplied by the Rate per cent. divided by the compound interest equals the annuity that £1 will buy.

To find what Annuity, or monthly sum, for any number of terms, a given sum will buy.

RULE. *Multiply the given Principal by the Annuity or the monthly payment that one Pound, or one Dollar will buy.*

EXAMPLE. What must be one of six equal annual payments to discharge a loan of £864,,10 for six years at 8%.

$$\frac{£1.5869\times.08\times864.5}{.5869}=£187.\quad \text{Ans.}\quad £187,,0,,0.$$

To find the cost of a given annuity, for any time. and Rate.

RULE. *Multiply the Present Worth of an Annuity of £1 for the given time and Rate by the given number of Pounds.*

EXAMPLE. Find the cost of an Annuity of £187 Stg. for six years, interest 8 per cent. per annum.

$$\frac{.5869\times187}{.08\times1.5869}=\ 864.5=\ £864,,10,,0.\quad \text{Ans.}$$

COMPOUND INTEREST TABLE.

TABLE showing the amount of one Pound sterling, or one Dollar at various rates per cent. for from one to twenty years, or other periodical terms.

This Table will serve as a model for making Compound Interest Tables for any number of terms, at any rate per cent. See Note, page 52.

No. of Terms.	1 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	10 per ct.
1	1.010000	1.050000	1.060000	1.070000	1.080000	1.100000
2	1.020100	1.102500	1.123600	1.144900	1.166400	1.210000
3	1.030301	1.157625	1·191016	1.225043	1.259712	1.331000
4	1.040604	1.215506	1.262477	1.310796	1.360489	1.464100
5	1.051010	1.276282	1.338226	1.402552	1.469328	1.610510
6	1.061520	1.340096	1.418519	1.500730	1.586874	1.771561
7	1.072135	1.407100	1.503630	1.605781	1.713824	1.948717
8	1.082856	1.477455	1.593848	1.718186	1.850930	2.143589
9	1.093685	1.551328	1.689479	1.838459	1.999004	2.357948
10	1.104622	1.628895	1.790848	1.967151	2.158924	2.593742
11	1.115668	1.710340	1.898299	2.104851	2.331638	2.853116
12	1.126825	1.795857	2.012197	2.252190	2.518169	3.138423
13	1.138093	1.885650	2.132929	2.409843	2.719623	3.452271
14	1.149474	1.979933	2.260905	2.578532	2.937193	3.797498
15	1.160969	2.078929	2.396559	2.759029	3.172169	4.177248
16	1.172578	2.182875	2.540351	2.952161	3.425942	4.594973
17	1.184303	2.292019	2.692772	3.158812	3.700017	5.054470
18	1.196146	2.406620	2.854339	3.379929	3.996018	5.559917
19	1.208107	2.526951	3.025599	3.616524	4.315699	6.115908
20	1.220188	2.653298	3.207135	3.869681	4.660955	6.727498

71 divided by the given rate will show the time in years in which any sum of money will double itself at Compound Interest; nearly.

The square of the amount of £1 for any given number of terms equals the amount for twice the given number of terms.

The cube of the amount of £1 for any given number of terms equals the amount for three times that number.

To prove interest: divide the computed interest by the interest for one day; the quotient should be the number of days in the example; or divide by the interest for one month; the quotient should be the number of months.

EQUATION OF PAYMENTS.

Equation of Payments is the process of finding the Equated Time, or the date when the sum of several debts due at different times may be paid.

Averaging Accounts is the process of finding the date on which the Balance is due.

Partial Payments are parts of a debt paid at different times; usually written on the back of notes and other interest bearing obligations, and called indorsements. The term also includes payments made on account of a debt before it is due.

The Term of Credit is the time to elapse before a bill becomes due.

The Average Term of credit is the time at the end of which the sum of several debts due at different dates may be paid at once.

The Equated Term is the average time for which interest is due on an account, or balance, and is always reckoned from the Zero Date.

The Zero Date is the date, or starting point, —from which all the other dates are reckoned; in this rule it is always the beginning—or starting point—of the month in which the first debt in the account occurs.

An Account is a statement of business transactions between Debtor and Creditor.

A Balance is the difference of two sides of an account.

A Cash Balance is the same, with the interest due.

The Creditor is entitled to *Interest* on the Balance from the date on which it is due to the date of settlement. The debtor is entitled to *Discount* off the Balance for the time he pays it before it is due.

BILLS BOUGHT ON UNEQUAL TIME ON THE SAME DATE.

1878, Jan. 1st,	Bought goods on 8 mos.	£100
" " "	" " " 6 "	100
" " "	" " " 7 "	100

On what date may the whole £300 be paid?

Term of Cr Mo..	Date.	
8	Jan. 1.	$100\times8=800$
6	" "	$100\times6=600$
7	" "	$100\times7=700$
		300)2100(7 mo. fr. Jan. 1, or Aug. 1.

Under the terms of this transaction the Debtor is entitled to the use of

1st, £100 for 8 months, =8 times 100 or £800 for 1 mo.
2d, 100 " 6 " =6 " 100 " 600 " 1 "
3d. 100 " 7 " =7 " 100 " 700 " 1 "

a credit equal to £2100 for 1 month ; this will evidently entitle the debtor to the use of £300 for as many months as 300 is contained in 2100.

The *product* of any number of *pounds* multiplied by any number of months, and fractions of a month, a Debtor is entitled to use them, is the number of pounds he is entitled to use for 1 month under the same terms, hence the following;—

Rule. Multiply each debt by its term of credit, divide the sum of the products by the sum of the debts, and the quotient is the equated term.

First study this very simple example thoroughly, make yourself familiar with each operation, the reason for its use, and the causes of the results, and you will then have no difficulty in comprehending the most complex Debtor and Creditor accounts.

BILLS BOUGHT ON EQUAL TIME AT DIFFERENT DATES.

Required the equated time of paying the following bills each bought on 8 months credit.

No of months, from zero date	1878 June	0—Zero date.		
	June	9	£180 × .3 =	54
1	July	15	84 × $1\frac{1}{2}$ =	84 42
3	Sept.	14	240 × $3.3\frac{1}{6}$ =	720 72 40
4	Oct.	10	96 × $4\frac{1}{3}$ =	384 32
			£600	)1428(2.38
				3
				11.4

	mo. da.		yr. mo. da.	
Equated term	2, 11	after	78, 6, 0	zero date.
Plus term of Cr.	8, 0	=	10, 11,	
Equated time			79, 4, 11,	or April 11th, 1879.

Rule, Multiply each debt by the time, in months and fractions of a month, between its occurrence and the zero date, divide the sum of the products, by the sum of the debts, and the quotient is the equated term— in months and hundredths of a month,— counting from the zero date, add the term of credit, and the sum is the equated time.

NOTE 1. To reduce hundredths of months to days, multiply by 3, and point off the right hand figure, when the right hand figure in the product is 5 or more add 1 day, otherwise disregard it.

NOTE 2, When the figures representing the day of the month are multiples of 3, such as the 3d, 9th, 27th, &c. &c., multiply by tenths, because 3 days is .1 of a month ; when they are not multiples of 3 then multiply by the simplest fraction, or fractions of a month. In the above example, Sept. 14th,. 3 months 14 days from zero date, we multiply by $3.3\frac{1}{6}$, 3 months, plus 9 days, plus 5 days. Facility in selecting the simplest fractions for multipliers is easily acquired by practice.

BILLS BOUGHT ON UNEQUAL TIME AT DIFFERENT DATES.

Required the equated time of paying the following bills of goods.

Term of Cr. Mos.	April	0			
6	"	10	To Mdse.	£310×$6\frac{1}{3}$ =	1860, 103
2	1 May	21	" "	468×3.7 =	1404, 328
4	2 June	1	" "	520×$6\frac{1}{30}$ =	3120, 17
3	3 July	8	" "	750×$6.1\frac{1}{6}$ =	4500, 75, 125
				2048	)11532(5.63

	Mo.	Da.	
Zero date	4	0	
Equated term	5	19	3
Equated time	9	19 or Sept. 19th.	18.9

Rule- Multiply each debt by the term of credit, plus the time between the date of the transaction and the zero date; divide the sum of the products by the sum of the debts, and the quotient is the equated term.

The figures on the extreme left represent the terms of credit; the figures on the left of the month represent the number of months from the zero date, these together with the day of the month are the multipliers.

	mos.		mos. da.		
1st item	6	Cr. plus	0, 10	from 0 date	=$6\frac{1}{3}$ mos.
2d "	2	" "	1, 21	" " "	=3.7 "
3d "	4	" "	2, 1	" " "	=$6\frac{1}{30}$ "
4th "	3	" "	3, 8	" " "	=$6.1\frac{1}{6}$ "

NOTE. The use of the beginning of the month, instead of the date of the first transaction for the starting point, makes no difference in the ultimate result, and avoids the continual labor of finding on each item, the time between two dates, each date as written, *itself* representing the time.

AVERAGING ACCOUNTS.

Find the equated time of paying the balance of the following accts

1878			Dr.		1878			Cr.	
Mar.	0				Mar	0			
"	15	3 mos.	600×3½=	1800 300	2 May	10	By Cash	300×2⅓ =	600 100
1 Apr.	3	4 "	700×5.1=	3500 70	4 July	1	" "	400×4⅓₀=	1600 13
2 May	10	6 "	1000×8⅓=	8000 333	5 Aug	15	" "	500×5½ =	2500 250
			2300	14003				1200	5063
			1200	5063					
			1100	)8940(8.13					
				3					
				3.9					

Balance due Nov. 4th, 8 mos. 4 days after zero date.

1877			Cr.		1877			Dr.	
June	0				June	0			
1 July	4	By Note	158×1.1⅓₀=	158 16 5	"	20	To Goods	986×⅓ =	329 329
5 Dec.	18	" Md'e	228×6.6 =	1368 137	5 Nov.	16	" "	152×5⅓⅕=	760 51 30
1878					1878				
9 Mar	5	" "	450×9⅙ =	4050 75	8 Feb.	26	" "	110×8.7⅙=	880 77 18
			836	5809				1248	2474
				2474				836	
				412)3335(8.09				412	
				3					
				2.7 8 mo. 3da before zero date.					

Zero date 77, 6, 0
Minus 8, 3

76, 9, 27 Balance due Sept. 27th, 1876.

NOTE. - The Dr. and Cr. sides are here transposed for convenience.

In this example the balance of the products is on the smaller side of the account; when this happens the equated term is deducted from the zero date to find the equated time.

The credit side has the ADVANTAGE of the use of the equivalent of £3335 for one month, then the other side is entitled to interest on the balance for as many months as 412 is contained in 3335.

Rule. Multiply each item by the time between its occurrence and the zero date, added to the term of credit—if any— divide the balance of the products by the balance of the account and the quotient is the equated term.

CASH BALANCES.

By the use of the following Rule the final desired result, the *Cash Balance,* may be found in less time than is required to find the *Date* on which the Balance is due.

RULE.—*Multiply each item by the number of months and fractions of a month between its date and the date on which the Cash Balance is required; the difference of the sums of the Products* ×.01 *is the interest on the Balance at one per cent. per month.* See note page 46.

Find the Cash Balance on the lower example page 70, March 5th, 1878; interest 1% per month. Ans. $483.13.

1877		*Dr.*		
June	20	To Goods.	$986 × 8\frac{1}{2}$ =	7888
Nov.	16	"		493
1878				456
Feby	26	"	152 × 3.3$\frac{1}{3}$ =	46
				51
			110 × .3 =	33
			$1248	8967
			836	1854
		Balance	$412	Interest $71.13

1877		*Cr.*		
July	4	By Cash	$158 × 8$\frac{1}{30}$ =	1269
Dec.	18	"		456
1878			228 × 2.4$\frac{1}{6}$	91
				38
Mar.	5	"	450	
			$836	1854

MONTHLY STATEMENTS.

Find the Cash Balance on the following account, at the end of the month, Interest 6% per annum. Ans. $5345.73.

Jany.	3	To Goods.	$841.28 × .9	=	757.1
"	5	"	730.75 × $\frac{1}{2}$ & $\frac{1}{3}$	=	609.0
"	6	"	815.00 × .8	=	652.0
"	10	"	660.00 × $\frac{2}{3}$	=	440.0
"	15	"	786.20 × $\frac{1}{2}$	=	393.0
"	18	"	1000.00 × .4	=	400.0
"	27	"	496.00 × .1	=	49.6
			$5329.23	Int at 12%	$33.007
$33.00 ÷ 2 = Int. at 6%. =			16.50		
			$5345.73		

Interest at 1% per month is equal to 12% per annum; interest at 12% per annum ÷ 4 = 3%; ÷ 3 = 4%; × $\frac{5}{12}$ = 5%; ÷ 2 = 6%, × $\frac{7}{12}$ = 7%; × $\frac{5}{8}$ = 7$\frac{1}{2}$%; − $\frac{1}{3}$ = 8%; − $\frac{1}{4}$ = 9%; etc.

PARTIAL PAYMENTS.

Bankers make a Business of loaning Money for Profit; they therefore find the *Amount* due at the date on which a Payment is made, deduct the Payment, and regard the *Balance* as a new *Principal.*

Merchants and Traders charge and allow the same Rate of Interest on both sides of the account.

BANKERS' RULE AT ONE PER CENT. PER MONTH INTEREST.

RULE. *Multiply the Principal by the number of months and fractions of a month between its date and the date of a Partial Payment, add the product* × .01 *to the Principal and deduct the Payment; the Balance is the new Principal.*

A note is made for £1000 Stg. March 3rd, 1882, endorsed May 15th, £100, July 27th, £200, find the Cash Balance March 3rd, 1883. Ans. £799,,18,,0.

£1000.00 × 2.4 × .01 = 24.00. 1000 + 24.00 — 100 = 924 = Bal. May 15 £924,,0,,0
924.00 × 2.4 × .01 = 22.176. 924 + 22.176 — 200 = 746.176 = Bal. July 27, 746,,3,,6
746.176 × 7.2 × .01 = 53.724 746.176 + 53.724 = 799.9 = Cash Bal. Mar. 3, 799,,18

MERCHANTS AND TRADERS' RULE AT ONE PER CT. PER MONTH.

RULE. *Multiply each item by the number of months and fractions of a month between its date and the date of settlement; the Balance of the Products* × .01 *is the Interest on the Balance.*

Find the Cash Balance on the above Note, Merchants and Traders Rule. Ans. £796,,0,,0.

May 15th,	100 × 9.6 = 960		1000 × 12 =	12000
July 27th,	200 × 7.2 = 1440		300	2400
	300	2400	Bal. 700 Int. on Bal.	9600

£700 + 96 = £796,,0,,0 Cash Balance March 3rd, 1883.

MERCHANTS AND TRADERS' RULE AT FIVE PER CT. PER ANNUM.

RULE. *Multiply each item by the number of days between its date and the date of settlement; divide the Balance of the Products by* 3; *to the Balance add the Quotient plus* .1 *plus* .01, *and remove the decimal point four places to the left.*

Find the Cash Balance on the same acct. interest 5%

May 15th,	100 × 292 = 29200	.1000 × 365 = 36.5000
July 27th,	200 × 219 = 43800	7.3000
	73000	3)29.2000
		9.7333
		.9733
		.0973
		40.0040 *

£700 + 40 = £740,,0,,0. Ans.

* See page 50.

COMMERCIAL DISCOUNT is a given rate per cent. allowed off a *Debt*, or part of a *Debt*, not off the money paid, in consideration for Cash.

The terms of the following transaction are 6 months' credit; 7% discount for cash in ten days; 6% in one month; 4% in two months.

A discount of 7% enables £93 to pay £100, consequently every £100 paid discharges £107.5269 of the Debt; discounts at the other rates named result in like manner.

GEORGE SMITH & SONS.

Dr. *In account with* BROWN & CO. *Cr.*

1883.				1883.			
Jan....14	To Mdse. 6 Mo.	2,147	583	Jan....24	By Cash	1,000	
					„ Discount 7 p. c.	75	269
				Feb. . 13	„ Cash	500	
					„ Discount 6 p. c.	31	915
				Mch.. 13	„ Cash	300	
					„ Discount 4 p. c.	12	5
				July . 14	*Bal. due without discount*	227	899
		2,147	583			2,147	583
July...14	To Balance.........	227	899				

On what date is the following balance due? Rule, page 70.

1883.	*Dr.*		1883.	*Cr.*	
Jan. ...1	To goods, 6 mos....	1500	March 1	By Cash	300
			May.... 1	„ „	400
				Balance	800
Jan. ...1	1500 × 6 = ...	9000	March 1	300 × 2 = ...	600
		2200	May... 1	400 × 4 = ...	1600
		800)6800			2200

Ans.—$8\frac{1}{2}$ months after Jan. 1st,—Sept. 15th.

EXPLANATION. Under the terms of this transaction the debtor is entitled to the use of $1500 for 6 months, equal to 6 times 1500 or $9000 for 1 month on paying
$300 in 2 months, the use of which for that time is = $600 for 1 month
400 „ 4 „ „ „ „ „ „ „ „ „ = 1600 „ 1 „
he has used the equivalent of $2200 for 1 month, and is consequently entitled to the use of the balance for a time equal to the use of $6800 for one month.

CASTING OUT THE NINES.

The number nine has many peculiar properties in our system of notation. Any number is divisible by nine when the sum of its digits is divisible by nine.

Any remainder left after dividing a number by 9, will be left after dividing the sum of its digits by 9.

This peculiarity may be used with advantage in proving the four fundamental rules, by casting out the nines; that is, dropping whenever the sum reaches or exceeds that number; thus to cast the 9s out of 846732, we say 8+4 less 9 leaves 3; 3+6 less 9 leaves 0; 7+5 less 9 leaves 3; hence the following.

To prove ADDITION, cast out the nines from the example, and from the ascertained sum; if correct the excess in each will be the same.

To prove SUBTRACTION, the excess of the remainder should equal the excess in the minuend less the excess in the subtrahend.

Note. If the excess in the minuend is less than the excess in the subtrahend, it must be increased by nine.

To prove MULTIPLICATION. The excess of the product must equal the product of the excesses of the factors.

Note. If the multiplier or multiplicand is a multiple of nine, the product will have no excess.

To prove DIVISION. The excess of the dividend must equal the product of the excesses in Quotient and Divisor plus the excess of the remainder.

Subtraction, by Addition, by the use of the Number Nine.

RULE. Write nine times the subtrahend under the minuend, add each figure of the upper number to the figure of the same order, and all the inferior places, of the lower number, carrying as in addition, and stopping at the last carrying figure.

RAPID RULES FOR FARMERS.

The practice of buying or selling grain by the 100 pounds, or the *cental* system, is becoming almost universal, and has many advantages over the old practice of selling grain by the bushel.

The following rules for finding the relative values of the bushel and the cental are easy to learn, and true and rapid in execution.

To find the value per cental when the price per bushel is given.

RULE. Set down the price per bushel; remove the decimal point two places to the right, and divide by the number of pounds in the bushel.

EXAMPLE. If wheat is $1.80 per bushel, what is its value per cental? $\therefore\ \frac{180}{60}=3$. Ans. $3.

To find the value per bushel when the price per cental is given.

RULE. Set down the price per cental; multiply by the number of pounds in the bushel, and remove the decimal point two places to the left.

In dealing with English market quotations write the given price per cental in pence, and divide by 20, the value per bushel will be in shillings. $\frac{1}{10}$ or ·1 the price per cental in *pence* $\times$ 4 = the price per quarter in shillings.

EXAMPLE. If wheat is quoted at 8s. 9d. per cental, what is the value of a bushel?

$$8s.\ 9d.=105d.\quad \frac{105\times 60}{100}=63d., \text{ or } \frac{105}{20}=5{\cdot}25=5s.\ 3d.$$

The price per cental in U.S. Dollars, multiplied by 4·11, equals the value per cental in English shillings, thus: wheat at $3 per cental=4·11 × 3=12·33=12s. 4d.

NOTE. The number of pounds estimated to the bushel must conform to the local usage; in the above examples the bushel is assumed to be equal to 60 lbs. When 63lbs. = one bushel increase 5 per cent.

To find the number of cubic feet in a Hay Stack.

If the Stack is round, add the height to the eaves—in feet to $\frac{1}{3}$ the height from the eaves to the top, Multiply this sum by the square of the diameter Multiplied by .7854; or Multiply by the square of the circumference Multiplied by .07958.

If the Stack is square find the height in the same way and Multiply the height by the square of one side.

If the Stack is rectangular with gable ends add the height to the eaves to $\frac{1}{2}$ the height from the eaves to the top, Multiply the sum by the length of the stack Multiplied by the width.

The number of cubic feet to be reckoned for a ton, depends upon the character of the hay and local usage.

RAPID RULE FOR RECKONING THE COST OF HAY.

RULE. *Multiply the number of pounds by half the price per ton, and remove the decimal point three places to the left.*

EXAMPLE What is the cost of 764 lbs. of hay at $14 per ton?

$$\overline{764\times 7}\div 1000=5.348.$$

NOTE. The above rule applies to anything of which 2,000 pounds is a ton.

To find the number of trees required to plant an acre.

RULE. Divide 43560 by the number of square feet occupied by one tree.

The trees being eight feet apart, how many are required to plant an acre?

43560ft. = 1 acre. $\frac{43560}{8\times 7}=778$ trees. Ans.

TO MEASURE GRAIN.

RULE. Level the grain; ascertain the space it occupies in cubic feet; multiply the number of cubic feet by 8, and point off one place to the left.

EXAMPLE. A box level full of grain is 20 feet long, 10 feet wide, and 5 feet deep. How many bushels does the box contain? Ans. 800 bush.

Process $20 \times 10 \times 5 \times 8 \div 10 = 800.$

Or,

$$\begin{array}{r} 1000 \text{ ft.} \\ 8 \\ \hline 800.0 \end{array}$$

Note. Exactness requires the addition of 44.5 bushels to every ten thousand U. S. Bushels.

Cubic feet$\times$.779=Imperial Bushels nearly.

The foregoing rule may be used for finding the number of gallons, by multiplying the number of bushels by 8.

If the corn in the box is in the ear, divide the answer by 2, to find the number of bushels of shelled corn, because it requires two bushels of ear corn to make one of shelled corn.

RAPID RULES FOR MEASURING LAND WITHOUT INSTRUMENTS.

In measuring land, the first thing to ascertain is the contents of any given plot in square yards; then, given, the number of yards, find out the number of rods and acres.

The most ancient and simple measure of distance is a step. Now, an ordinary-sized man can train himself to cover 1 yard at a stride, on the average, with sufficient accuracy for ordinary purposes.

To make use of this means of measuring distances, it is essential to walk in a straight line; to do this, fix the eye on two objects in a line straight ahead, one comparatively near, the other remote; and in walking, keep these objects constantly in line.

Farmers and others by adopting the following simple and ingenious contrivance, may always carry with them the scale to construct a correct yard measure.

Take a foot rule, and commencing at the base of the little finger of the left hand, mark the quarters of the foot on the outer borders of the left arm, pricking in the marks with indelible ink.

To find the area of a four-sided figure, the opposite sides being parallel

RULE. Multiply the length and the breadth together, and the product is the area.

To find the area of a square, square one of its sides.

When the length of two opposite sides is unequal, add them together, and take half the sum and multiply by the breadth.

EXAMPLE 1. How many square yards in a square piece of land, 101 yds. on each side?

Process $101^2 =$ Ans. 10,201 yards.

EXAMPLE 2. How many yards in a piece of land 60 yards long and 20 yards wide? Ans. 1200.

Process $600 \times 2 = 1200$.

Only a sufficient number of examples to clearly illustrate the working of the Rules are presented; the Teacher or Student may furnish additional examples for exercises.

EXAMPLE 3. How may yards in a piece of land, one side is 40 yards long, and the other side 60 yards long, parallel sides being 10 yards apart?

Process, $$\frac{40 + 60 \times 10}{2} = 500.$$

500 yards, Ans.

To find the area of any three-sided figure.

RULE. Multiply the longest side into one-half the distance from this side to the opposite angle.

EXAMPLE. What is the area of a triangular plot of land, the longest side of which is 80 yards, and the shortest distance from this side to the opposite angle 40 yards?

Process, $$\frac{40 \times 80}{2} = 1600 \text{ yds. Ans.}$$

To find how many rods in length will make an acre, the width being given.

RULE. Divide 160 by the width, and the quotient will be the answer.

EXAMPLE. If a piece of land be 4 rods wide, how many rods in length will make an acre?

$$160 \div 4 = 40 \text{ rods Ans.}$$

NOTE. In measuring irregular plots of land divide it into rectangles and triangles, and take the *sum* of the measurements.

To find the number of acres in any plot of land, the number of rods being given.

RULE. Divide the number of rods by 8, and the quotient by 2, and remove the decimal point one place to the left.

EXAMPLE. In 6840 rods, how many acres?

$42\frac{3}{4}$ acres Ans.

Process.

$$\begin{array}{r} 8)6840 \\ \hline 2)855 \\ \hline 42.75 \end{array}$$

To find the number of acres, the number of yards being given.

Divide the number of yards by 4840 or its factors.

EXAMPLE. Find how many acres in 21,780 yds.

$$\frac{21,780}{10\times11\times11\times4}=4.5 \qquad \text{Ans. } 4\tfrac{1}{2} \text{ acres.}$$

A circle encloses the largest area within the shortest fence.

The length of a circular fence = the square root of the area × 3·545

Find the length in yards of a circular fence to enclose 10 acres.

$\sqrt{48400}=220.\quad 220\times3{\cdot}545\ =780$ yards.

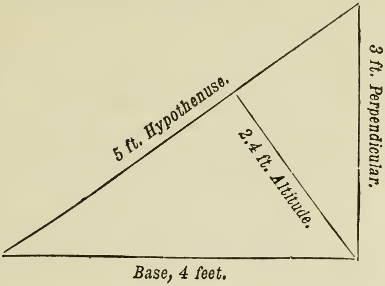
A square plot of the same area requires a fence 880 yards long.

The largest area enclosed within the shortest fence, in a rectangular plot, is a square.

$393\frac{1}{2}$ yards of fence will enclose a square plot of two acres; it would require 2 miles and 2 rods of fence to enclose the same area in a rectangular plot 1 rod wide.

RAPID RULES FOR MECHANICS

To lay off a square corner. Take a measure and lay off with it a triangle, one side of which is four ft long, another three feet, and the remaining side five ft., this triangle will be right-angled, and the two shorter sides will serve to lay off an exact square.



TRIANGLES. The *Area*=the *Base*× half the *Perpendicular*.

The *Area*=√of the product of half the sum of the three sides×by the three remainders of each side subtracted from the half sum.

The *Hypothenuse*=√of the sum of the squares of the *base* and the *perpendicular*.

Or, divide the square of the *Base* by the sum of the *Hypothenuse* and the *Perpendicular*. Half the sum of the Divisor and the Quotient, equals the *Hypothenuse*.

A Diagonal line from the upper, to the opposite lower corner of a room = the square root of the sum of the squares of the length, breadth and height of the room.

Either *Base* or Perpendicular=√of the difference of the squares of the *Hypothenuse* and the given side.

The *Altitude* = twice the quotient of the *area* ÷ the given *base*, or the *Hypothenuse* being the base, divide the product of the two other sides by the *Hypothenuse*.

The number of board feet in a *Telegraph pole*, or any frustrum of a Pyramid,=four times the sum of the areas of the two ends and the mean in feet × the height.

The number of board feet in a wedge = twice the sum of the three parallel edges, in feet × the width of the *Butt* × the length.

The *Area* of a square constructed upon the Hypothenuse of a triangle is equal to the sum of the areas of the squares constructed upon the other two sides.

A WEDGE, the solidity$=\frac{1}{6}$ the sum of the three parallel edges $\times$ by the breadth of the butt $\times$ by the length; or $\frac{1}{3}$ the sum of the area of the Base and $\frac{1}{2}$ the product of the feather edge, and the other side of the Base $\times$ the length.

The *Frustrum* of a *Cone*, a *Pyramid*, or a *Wedge*, the *Solidity*$=\frac{1}{3}$ the sum of the areas of the two ends and the mean proportional $\times$ the height.

The *Mean Proportional* of the Frustrum of a Cone, or the Frustrum of a Pyramid=the product of the diameters of the two ends; of the Frustrum of a Wedge$=\frac{1}{2}$ the sum of the products of either of the different edges of the butt$\times$the other edge of the top.

The height of a Pyramid=the height of its Frustrum$\times$ the diameter of the Base$\div$the difference of the two end diameters.

To find the number of BOARD feet in a telegraph pole, the diameters being given in inches, and the height in feet.

If the two ends are square, multiply the sum of the squares of the two end diameters, plus the product of the two end diameters by $\frac{1}{3}$ the height, and divide by 12. If the pole is round, multiply the squares of the two end diameters, plus the product of the two end diameters, by the height$\times$by.0218. For cubic feet$\times$.001818.

ACOUSTICS. The time, in seconds, multiplied by 1125, gives, in feet, the distance of sound.

GRAVITY. The square of the number of seconds$\times$ $16\frac{1}{12}$=the distance, in feet, a body will fall in a given time.

MOMENTUM. The weight, in pounds$\times$the velocity in feet, per second, gives the momentum of bodies.

ATMOSPHERE. The weight of the atmosphere, in pounds, at the surface of the ocean=the given area, in square inches$\times$15.

WATER POWER. The weight or pressure, in pounds, of water at any given depth, on a square foot=the depth, in feet$\times 62\frac{1}{2}$.

THE LATERAL pressure, in pounds=the area of the reservoir,$\times$half its average depth$\times 62\frac{1}{2}$

To measure rectangular surfaces.

RULE. *Multiply the length by the breadth.*

How many square feet in a floor 18 ft. wide, and 20 ft. long?

$18 \times 20 = 360.$ Ans. 360 ft.

How many square ft. in a board $1\frac{1}{2}$ ft. wide, 21ft. long?

$\frac{21 \times 18}{12}$ or $\frac{21 \times 3}{2} = 31\frac{1}{2}.$ Ans. $31\frac{1}{2}$ ft.

A board being 4 inches wide, how many inches in length will be equal to a square foot?

$\frac{144}{\text{the width.}}$ thus $\frac{144}{4} = 36.$ Ans. 36 inches.

To measure a board wider at one end than the other of a true taper.

RULE. *Add the widths of the two ends, halve the sum, and multiply by the length.*

How many square feet in a board 20 feet long, 9 inches wide at one end, and 11 in. at the other?

$\frac{9+11}{2} = 10$ in. mean width $\frac{20 \times 10}{12} = 16\frac{2}{3}$ sq. feet. Ans.

To find the *Board Measure* of planks and joists.

NOTE.—A Board foot = 1 square foot, 1 inch in thickness.

RULE. *Multiply the length in feet by the product of the thickness and the width in inches, and divide by* 12.

How many board ft. in a plank 18ft. long, 10in. wide, and 4 in. thick?

$\frac{18 \times 10 \times 4}{12}$ or $\frac{6\ \cancel{18} \times 10 \times 4}{3\ \cancel{12}} = 60$ feet. Ans.

To find how many board feet, one inch in thickness, can be sawed from a round log of any given length.

RULE. *Subtract four inches from the given diameter, square the difference, Multiply by the length in feet, and divide by* 16.

How many board feet can be cut from a log 24 inches in diameter, 18 feet long?

$24 - 4 = 20$ $\frac{20 \times 20 \times 18}{8 \times 2} = 450$ feet. Ans.

To find the cost of any number of feet of Lumber.

RULE. *Multiply the given number of feet by the price per* ·1000, *and remove the point three places to the left.*

To find the number of cubic feet in a round log.

RULE 1. *Divide the mean girt by 3·545, and multiply the square of the quotient by the length of the log.*

RULE 2. *Multiply the square of the mean girt by the length multiplied by* ·08.

RULE 3. *Square ¼ the mean girt, and multiply by the length.*

How many cubic feet in a log 10 ft. long, girt 8 ft.?

$$\frac{8}{3{\cdot}545} = 2{\cdot}257 \qquad 2{\cdot}257^2 \times 10 = 50{\cdot}92 \text{ feet.}$$

$$8^2 = 64 \qquad 64 \times 10 \times {\cdot}08 = 51{\cdot}2 \quad \text{,,}$$

$$\frac{8}{4} = 2 \qquad 2^2 \times 10 = 40 \quad \text{,,}$$

How many cubic ft. in a log 10 ft. long, girt 12·6492?

$$\frac{12{\cdot}6492}{3{\cdot}545} \qquad 3{\cdot}5682^2 \times 10 = 127{\cdot}321 \text{ ft.}$$

$$12{\cdot}6492^2 \times 10 \times {\cdot}08 = 128 \quad \text{,,}$$

$$\frac{12{\cdot}6492}{4} \qquad 3{\cdot}1623^2 \times 10 = 100 \quad \text{,,}$$

NOTE.—Rule 1 is the true method; Rule 2 gives an excess of ·00533, about ½ of 1 per cent., or 5⅓ feet to each 1000; Rule 3 is common in England, but untrue, the advantage in favor of the buyer being 27·321 feet in excess of every 100 feet he pays for; equal to a discount of 21½ per cent.

To find the number of feet superficial in a case.

RULE. *Multiply the width by twice the depth, then multiply the sum of the depth and width by twice the length, and add the two products.*

How many superficial feet in a case $3\frac{1}{2} \times 4\frac{1}{4} \times 6$ ft.?

$4\frac{1}{4} \times 7 = 29\frac{3}{4}$. $\quad 7\frac{3}{4} \times 12 = 93$. $\quad 29\frac{3}{4} + 93 = 122\frac{3}{4}$ ft. Ans.

Baltic and American deals, battens and planks are sold in the English Market by the St. Petersburgh Standard.

The St. Petersburgh Standard = 120 pieces $12 \times \frac{3}{24} \times \frac{11}{12}$ ft. = 165 cubic feet.

1 Standard = 3·3 loads of 50 feet. Cubic feet $\times \cdot 00\dot{6}0\dot{6}$ = Standards, thus

$$\left.\begin{array}{lll} 1200 \text{ pieces} & 12 \times \frac{1}{4} \times \frac{3}{4} & = 2700 \\ 1200 \quad \text{,,} & 12 \times \frac{1}{4} \times \frac{7}{12} & = 2100 \\ 1200 \quad \text{,,} & 12 \times \frac{5}{24} \times \frac{13}{24} & = 1625 \end{array}\right\} = 6425.$$

$$6{\cdot}425 \text{ feet} \times \dot{6}0\dot{6} = 38{\cdot}935 \text{ standards.}$$

Much mental effort and the use of many figures are saved by writing all the terms in the denomination, or the largest possible fractions of the denomination required in the answer: *i.e.* in feet or fractions of a foot, yds. or fractions of a yard, pounds or fractions of a pound, &c.

To find how many solid feet a round stick of timber will contain when squared; the smallest diameter being given.

RULE. *Multiply the length by the Product of the diameter and the Radius, all in feet.*

Find how many solid feet when squared, in a round log $2\frac{1}{2}$ feet wide and 10 feet long.

$$\frac{5\times5\times10}{2\times4}=31.25 \text{ feet. Ans.}$$

General rule for measuring timber to find the solid contents in feet.

RULE. Multiply the depth, in feet, or fractions of a foot, by the breadth, multiplied by the length.

How many solid feet in a piece of timber 2 feet wide, 10 inches thick and 12 feet long.

$$\frac{2\times5\times12}{6}=20 \text{ feet.}$$

To find the contents of a true tapered pyramid, whether round, square, or triangular.

RULE. Multiply the area of the base by $\frac{1}{3}$ the height.

How many cubic feet in a round stick of timber, truly tapering to a point, $1\frac{1}{2}$ feet in diameter at the base and 24 feet long.

$$\frac{3\times3\times22\times8}{4\times4\times7}=14.14+ \text{ feet.}$$

How many cubic feet in a square block of marble, truly tapering to a point, 24 inches on each side at the base, and twelve feet high.

$$\frac{24\times24\times4}{144} \text{ or } 2\times2\times4=16 \text{ feet, Ans.}$$

The diameter being given, to find the circumference.

RULE. *Multiply the diameter by* $3\frac{1}{7}$.

EXAMPLE. What is the circumference of a wheel the diameter of which is 42 inches? Ans. 11 ft.

$$\frac{42\times 3\frac{1}{7}}{12} \text{ or } \frac{7\times 22}{2\times 7}=11 \text{ feet.}$$

To find the diameter when the circumference is given.

RULE. *Divide the circumference by* $3\frac{1}{7}$.

EXAMPLE. What is the diameter of a wheel, the circumference of which is 11 feet? Ans. $3\frac{1}{2}$ feet.

Process $$\frac{\cancel{11}}{1}\times\frac{7}{2\ \cancel{22}}=3\frac{1}{2}$$

What is the width of a circular pond, 154 rods in circumference? Ans. 49 rods.

Process $$7\ \frac{\cancel{154}}{1}\times\frac{7}{\cancel{22}}=49.$$

The diameter being given, to find the area.

RULE. Multiply the square of the radius by $3\frac{1}{7}$.

Find the area of a circle 36 inches in diameter.

$$\frac{3\times 3\times 22}{2\times 2\times 7}=7.07 \text{ feet.}$$

The length of a cylinder is equal to the capacity $\div$the square of the radius$\div 3\frac{1}{7}$.

Find the depth of a circular cistern, 7 feet wide, containing 2400 Imperial gallons.

$$\frac{2400\times 4\times 2\times 2\times 7}{25\times 7\times 7\times 22}=9.97 \text{ feet.}$$

See notes p. 116. 1 cubic foot$=6\frac{1}{4}$ Imp. Gallons, nearly.

To find the volume of a Cylinder.

RULE. Multiply the square of the radius by the thickness, both in feet, or fractions of a foot, and the product by $3\frac{1}{7}$; or,

Multiply the square of the diameter by the thickness, both in inches, and divide by 2200, the answer is in cubic feet; or,

Multiply the square of the diameter by .7854, and that product by the length.

EXAMPLE. How many feet in a grindstone 24 inches in diameter and 4 inches thick?

1st Method. 2nd Method.

$$\frac{1\times1\times22}{3\times7}=1.04 \qquad \frac{24\times24\times4}{2\times11}=1.04 \text{ ft.}$$

A Cylindrical foot is the volume of a cylinder, one foot in depth and diameter, and is equal to 1728 cylindrical inches. Cylindrical inches, $\times$.7854=cubic inches.

A *Cylinder*, the surface = the circumference $\times$ the length. The capacity of a cylinder in Imperial Gals. = the square of the diameter $\times$ the length—all in inches— $\times$ ·0028325, for U.S. Gallons $\times$ ·0032; or if the terms are in feet $\times$ 4·9 or 4·89469 for Imperial Gals.; for U.S. Gals. $\times$ $5\frac{1}{7}$.

TO MEASURE FREIGHT, ETC.

RULE. *Multiply together the length, breadth and depth of one package—in feet, and the largest fractions of a foot—and multiply by the given number of packages of the same dimensions.*

Find the number of cubic feet in six packages, each 1ft. by 1ft. 2in. by $1\frac{3}{4}$ft.

$$\frac{1\times7\times7\times6}{6\times4}=12\tfrac{1}{4}\text{ft. Ans.}$$

Find the charges on 1000 cases, each $16\times12\times6$ inches, at 16s. per ton of 40ft.

$$\frac{1000\times4\times1\times1\times16}{3\times2\times40}= {}^{\cdot}266\tfrac{2}{3}\text{s.} = \pounds13\ 6\text{s. } 8\text{d.}$$

Bricklayers' Work.

The usual size of English bricks is $9 \times 4\frac{1}{2} \times 2\frac{1}{2}$ inches; U.S. bricks vary, usually $8 \times 4 \times 2$ inches; in estimating the number of bricks required to build a wall add half an inch to the thickness of the brick to allow for the thickness of the mortar.

The number of bricks required to build a wall = the length × the height × the width ÷ the dimensions of the bricks; or for English bricks multiply the length, height, and number of bricks thick by 11; the excess will be about 3 per cent. for breakage, &c.; for American bricks, multiply by 15.

How many English bricks are required to build a wall 78 feet long, 20 feet high, and $1\frac{1}{2}$ feet thick?

$$\frac{78 \times 20 \times 3 \times 4 \times 8 \times 4}{2 \times 3 \times 3 \times 1} = 33280 \text{ or } 78 \times 20 \times 2 \times 11 = 34{,}320$$

A bricklayer's hod measuring 1 ft. 4 in. × 9 in. × 9 in., equals 1,296 inches in capacity, and will contain 20 bricks, each $8 \times 4 \times 2$ inches.

A load of mortar measures 1 cubic yard, or 27 cubic feet; requires 1 cubic yard of sand, and 9 bushels of lime, and will fill 30 hods.

Plasterers' Work

Is measured by the square yard, for all plain work; by the foot, superficial, for plain cornices: and by foot, lineal, for enriched or carved mouldings in cornices.

Painters' Work

Is computed by the superficial yard; every part is measured that is painted, and an allowance is added for difficult cornices, deep mouldings, carved surfaces, iron railings, etc. Charges are usually made for each coat of paint put on, at a certain price per yard per coat.

SQUARE AND CUBE ROOT.

1. A square number multiplied by a square number, the product will be a square number.

2. A square number divided by a square number, the quotient is a square.

3. A cube number multiplied by a cube, the product is a cube.

4. A cube number divided by a cube, the quotient will be a cube.

5. If the square root of a number is a composite number, the square itself may be divided into integer square factors; but if the root is a prime number, the square cannot be separated into square factors without fractions.

6. If the unit figure of a square number is 5, we may multiply by the square number 4, and we shall have another square, whose unit period will be ciphers.

7. If the unit figure of a cube is 5, we may multiply by the cube number 8, and produce another cube, whose unit period will be ciphers.

8. If a supposed cube, whose unit figure is 5, be multiplied by 8, and the product does not give 3 ciphers on the right, the number is not a cube.

9. No even number not divisible by 4 is a perfect square.

10. No odd number which diminished by 1 is not divisible by 4, is a perfect square.

11. No number terminating in 2, 3, 7 or 8 is a perfect square.

12. A number ending in 5 cannot be a perfect square unless the number in the ten's place be 2.

To prove cube root: from a cube number subtract its root; the remainder will be a multiple of 6.

From a number that is not a cube, subtract the ascertained part of its cube root; divide the difference by 6; then divide the remainder in the example by 6; the excess, if any, should in each case be the same.

TABLE

For comparing the natural numbers with the unit figure of their squares and cubes. By the use of this, many roots may be extracted by observation:

Numbers...	1	2	3	4	5	6	7	8	9	10
Squares....	1	4	9	16	25	36	49	64	81	100
Cubes......	1	8	27	64	125	216	343	512	729	1000

The product of a number taken any number of times as a factor, is called a power of the number.

A root of a number is such a number as taken some number of times as a factor, will produce a given number.

If the root is taken twice as a factor to produce the number, it is the *square root;* if three times, the *cube root.*

By observing the above table, it will be seen that the square of any one of the digits is less than 100, and the cube of any one of the digits is less than 1000; therefore, the square root of two figures cannot be more than one figure.

The square of any number equals its root, plus the preceding square and root of a consecutive series.

$$4^2=16. \quad 4+9+3=16.$$

The units figure in the cube root of a perfect cube is the units figure in the *product* of the units figure of the cube multiplied twice into itself.

Find the cube root of 343.

The units figure $3\times3\times3=27$. Ans. 7.

The difference of the squares of two numbers equals their sum multiplied by their difference.

To find the square root of a number.

Use, to find the length of one side of a given square.

RULE 1. Separate the given number into periods of two figures each, beginning at the unit's place.

The number of figures in the root equals the number of periods.

2. Find the greatest number whose square is contained in the period on the left; this will be the first figure in the root. Subtract the square of this figure from the period on the left; to the remainder annex the next period to form a dividend.

3. Divide this dividend, omitting the figure on the right, by double the part of the root already found, and annex the quotient to that part, and also to the divisor; then multiply the divisor thus completed by the figure of the root last obtained, and subtract the product from the dividend.

4. If there are more periods to be brought down, continue the operation in the same manner as before until the last period has been brought down.

NOTE 1. If a cipher occurs in the root, annex a cipher to the trial divisor, and another period to the dividend, and proceed as before.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers, and continue the root to as many decimal places as are required.

Example. Find the square root of 643204.

```
         64'32'04 ( 802 Square Root.
         64
  1602)     3204
            3204
```

Given the Base and Perpendicular of a Triangle to find the Hypothenuse.

Double either Base or Perpendicular for a Divisor, square the other side for a Dividend, and proceed as for Square Root, the sum of the side doubled and the quotient is the answer.

Ex. Required the length of a ladder standing 80 feet from the Base, to reach the top of a cliff 798 feet high.

$\overline{798\times2}+4=1600$) 6400 (4 $\qquad$ $80^2=6400$.

6400 $\qquad$ $798+4=802$ ft. Ans.

To find the cube root of a number.

Use, to find the length of one edge of a given cube.

RULE 1. Beginning at the units' place, separate the given number into periods of three figures each; the number of figures in the root will be equal to the number of periods.

2. Find the greatest number whose cube is contained in the left-hand period; this will be the first figure in the root; subtract its cube, and to the remainder annex the next period.

3. Multiply the ascertained part of the root by 3, then multiply that result by the first figure in the root, the product with two ciphers annexed is the first trial divisor.

4. Find how many times the divisor is found in the dividend and place the result in the root, and also to the right of the first term in the left hand column; multiply the last result by the new figure in the root and add the product to the trial divisor; the sum is the complete divisor.

5. Multiply the complete divisor by the second figure in the root, subtract the product from the dividend and bring down the next period.

6. To find the next trial divisor add the square of the last found figure in the root to the preceding divisor and its smaller part; to the sum annex two ciphers, complete the divisor as before.

7. Repeat the foregoing process with each period until the exact root, or a sufficient approximation to it is found.

EXAMPLE. Find the length of one edge of an excavation from which a cubic mass of earth = 1,745,337,664 cubic feet is to be taken. Ans. 1204 feet.

32	300	1,745,337,664(1204, cube root.
	64	1
1st complete divisor,	364	745
	4,320,000	728
3604	14,416	17,337,664
2nd com. divisor.	4,334,416	17,337,664

NOTE 1. If a cipher occurs in the root, annex two ciphers to the trial divisor and another period to the dividend, and then proceed as before.

2. If there is a remainder, after the root of the last period is found, annex periods of ciphers and proceed as before to as many decimal places as the answer requires.

3. The cube root of a fraction may be found by extracting the cube root of the numerator and denominator, or reduce the fraction to a decimal and extract the root.

REFERENCE TABLES.

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4										
3	6	9									
4	8	12	16								
5	10	15	20	25							
6	12	18	24	30	36						
7	14	21	28	35	42	49					
8	16	24	32	40	48	56	64				
9	18	27	36	45	54	63	72	81			
10	20	30	40	50	60	70	80	90	100		
11	22	33	44	55	66	77	88	99	110	121	
12	24	36	48	60	72	84	96	108	120	132	144

Abbreviations used in Business.

A I, ...First Class.
@.........At.
℅ or Acc't ..Account.
Am't.........Amount.
Ass'd.........Assorted.
Bal...........Balance.
Bbl...........Barrel.
B. L..........Bill of Lading.
% ....Per cent.
Co............Company.
C. O. D. ..Collect on Delivery.
Cr............Creditor.
Com..........Commission.
Cons't........Consignment.
Cwt.......... Hundred Weight.
Dft............Draft.
Disc't........Discount.
Do............The same
Doz............Dozen.
Dr............Debtor.
E. E.........Errors excepted.
Ea...........Each.
Exch.........Exchange.
Exps.........Expenses.
Fol............Folio.
Fw'd..........Forward.
Fr't............Freight.
$, Dollar............C, Cents
Guar.........Guarantee.
Gal...........Gallon.
Hhd ....Hogshead.
Ins...........Insurance.
Inst..........This month.
Invt..........Inventory
Int............Interest.
Mdse.........Merchandise.
Mo............Month.
Net..........Without disc't.
No...........Number.
Pay't.........Payment.
Pd.............Paid.
Per An. .....By the year.
Pk'gs........Packages.
Per...........By.
£,,s,,d, Pounds, shil'gs, pence
Prem.........Premium.
Prox.........Next month.
Ps.............Pieces.
Rec'd.........Received.
R. R.........Railroad.
Ship't........Shipment.
Sund's.......Sundries.
S. S ....Steamship.
Ult...........Last month.

Specific Gravity is the weight of a body compared with another of the same bulk taken as a standard. The exact weight of a cubic inch of gold, compared with a cubic inch of water, is called its SPECIFIC GRAVITY. Water is the standard for solids and liquids.

A cubic foot of rain water weighs 1000 ounces Avoirdupois.

NOTE. To find the weight, in ounces, of one cubic foot of any substance here named, remove the decimal point three places to the right.

Air	·001
Alcohol, of Commerce	·835
„ Pure	·794
Alderwood	·800
Ale	1·035
Alum	1·724
Aluminum	2·560
Amber	1·064
Amethyst	2·750
Ammonia	·875
Ash	·800
Bees'-wax	·956
Blood, Human	1·054
Bone	1·660
Brass (about)	8·000
Brick	2·000
Butter	·942
Cherry	·715
Cider	1·018
Clay	2·160
Coal, bituminous (about)	1·250
„ anthracite	1·500
Copper	8·788
Coral	2·540
Cork	·240
Diamond	3·530
Earth (mean of the Globe)	5·210
Elm	·671
Emerald	2·678
Ether	·632
Fat of Beef	·923
Fir	·550
Flint	2·594
Glass plate	2·760
Gold, hammered	19·362
„ Coin	17·647
Granite	2·625
Graphite	1·987
Gunpowder	·900
Gum Arabic	1·452
Gypsum	2·288
Hazel	·600
Hematite Ore	4·705
Honey	1·456
Ice	·930
Iodine	4·948
Iridium	23·000
Iron	7·680
Iron Ore	4·900
Ivory	1·917
Lard	·947
Lead, cast	11·350
„ white	7·235
Lignum-vitæ	1·333
Lime	·804
„ stone	2·386
Mahogany	1·063
Malachite	3·700
Maple	·750
Marble	2·716
Men (Living)	·891
Mercury, pure	14·000
Mica	2·750
Milk	1·032
Naphtha	·700
Nickel	8·279
Nitre	1·900
Oak, very old and dry	·888
Oil, Castor	·970
Opal	2·114
Opium	1·337
Pearl	2·510
Pewter	7·471
Platinum Wire	21·041
Poplar	·383
Porcelain	2·385
Quartz	2·500
Resin	1·100
Salt	2·130
Sand	1·750
Silver coin	10·534
Slate	2·110
Steel	7·816
Stone	2·500
Sulphur	2·033
Tar	1·026
Teak	·745
Tallow	·941
Tin	7·291
Turpentine, spirits of	·870
Walnut	·671
Water, distilled	1·000
Wax	·897
Willow	·585
Wine	·992
Zinc, cast	7·190

The Diameter of a Circle.	$\times 3.1416$ $\div .3183$	=The Circumference.
	$\times .8862$ $\div 1.1284$	=The side of an equal Square.
	$\times .866$ $\div 1.1547$	=The side of an inscribed Equilateral Triangle.
	$\times .707$	=The side of an inscribed square
	$\times$ the Radius	=The area of do.
The Circumference of a Circle.	$\times .3183$ $\div 3.1416$	=The Diameter.
	$\times .2821$ $\div 3.545$	=The side of an equal Square.
	$\times .2756$ $\div 3.6276$	=The side of an inscribed Equilateral Triangle.
	$\times .2251$ $\div 4.4428$	=The side of an inscribed Square.
	$\times .15915$ $\div 6.28338$	=The Radius.
The Area of a Circle.	$\div 3.1416$	=The square of Radius.
	$\times 1.2732$ $\div .7854$	=The square of Diameter
	$\times 12.5663$ $\div .07958$	=The square of Circumference.

The Chord and Sine of an Arc or Segment of a Circle being given, the Diameter=the Sine, plus the quotient of the square of *half* the Chord÷the Sine.

The Surface of a Sphere, $=(\text{circumference})^2 \times .3183$.

The Volume of a Sphere =	Surface $\times$ 1-6 its Diameter. $(\text{Radius})^3 \times 4.1888$ $(\text{Diameter})^3 \times .5236$ $(\text{Circumference})^3 \times .0169$
The Diameter of a Sphere =	$\sqrt{}$ of Surface $\times .5642$ $\sqrt[3]{}$ of Volume $\times 1.2407$
The Circum. of a Sphere =	$\sqrt{}$ of Surface $\times 1.77255$ $\sqrt[3]{}$ of Volume $\times .38978$
The Radius of a Sphere =	$\sqrt{}$ of Surface $\times .2821$ $\sqrt[3]{}$ of Volume $\times .6204$

The Area of a Circle=$\frac{1}{2}$ the product of the circumference×the radius.

AN ELLIPSE. The area equals the product of the two diameters×.7854.

The Solid contents of any *Body*=the volume of water it displaces when immersed.

The height of any object=the length of its shadow × the height and ÷ the length of the shadow of any other object.

Longitude reckoned from the Meridian of Greenwich.

NORTH AND SOUTH AMERICA.

Place.	Lat.	Long.	Place.	Lat.	Long.
	° ′	° ′		° ′	° ′
Albany, N. Y..	42 40 N	73 45w	Lima, ...	12 3 s	77 6
AnnArb'r,Mich	42 17	83 43	Little Rock, Ark...	34 40 N	92 12
Annapolis, Md.	38 59	76 29	Louisville, Ky.	38 3	85 30
Augusta, Me...	44 19	69 50	Mexico, Mexico....	19 26	99 5
Austin, Texas.	30 13	97 39	Milwaukee, Wis...	43 2	87 54
Baltimore, Md.	39 18	76 37	Mobile, Ala........	30 41	88 1
Bangor, Me....	44 48	68 46	Montreal, C. E.....	45 31	73 33
Boston, Mass..	42 21	71 03	New Haven, Conn..	41 18	72 55
Brooklyn, N. Y.	40 42	73 58	New Orleans, La...	29 58	90 2
Buffalo, N. Y..	42 50	78 59	New York, N. Y...	40 43	74
Burlington, Vt.	44 27	73 10	Ottawa, C. W.......	45 23	75 42
Buenos Ayres.	34 36 s	58 22	Philadelphia, Pa...	39 57	75 9
Cambr'ge, Mass	42 23 N	71 08	Petersburg, Va....	37 14	77 24
Cape May, N. J.	38 56	74 57	Portland, Me.......	43 39	70 15
Cape Horn.....	55 59 s	67 16	Providence, R. I...	41 49	71 24
Charleston, S. C	32 47 N	79 56	Quebec, C. E......	46 40	71 12
Chicago, Ill....	41 54	87 38	Richmond, Va.....	37 32	77 26
Cincinnati, O...	39 06	84 30	Rochester, N. Y...	43 8	77 51
Columbia, S. C.	34	81 02	Rio Janeiro........	22 56 s	43 9
Concord, N. H.	43 12	71 29	Savannah, Ga......	32 5 N	81 5
Council Bluffs.	41 30	95 48	Sacramento, Cal...	38 35	121 28
Des Moines, Io.	41 35	93 40	St. August'e, Fla..	29 48	81 5
Detroit, Mich..	42 20	83 2	St. Louis, Mo......	38 37	90 15
Dover, Del.....	39 10	75 30	St. Paul, Minn.....	44 53	95 5
Dubuque, Io...	42 30	90 40	Salt Lake City.....	40 46	112 6
Fred'csb'rg, Va	38 18	77 27	San Francisco.....	37 48	122 47
Fort Laramie..	42 12	104 48	Santa Fe, N. Mex..	35 41	106 1
Ft. L'v'wth, Ks.	39 21	94 44	Springfield, Ill.....	39 48	89 33
Frankfort, Ky..	38 14	84 40	St. Joseph's, Mo...	39 40	94 52
Galveston, Tex	29 18	94 47	Syracuse, N. Y....	43 3	76 9
Georgetown,			Toronto, C. W.....	43 31	79 23
Bermuda, W. I.	32 22	64 37	Trenton, N. J......	40 13	74 45
Guayaquil......	2 13 s	79 53	Troy, N. Y.........	42 44	73 41
Havana	23 9 N	82 21	Valparaiso, ...	33 2 s	71 41
Halifax.........	44 39	63 35	WASHINGTON	38 53 N	77 0
Harrisburg, Pa.	40 16	76 50	West Point, N. Y..	41 23	73 57
Hartford, Conn	41 46	72 41	Wheeling, W. Va..	40 7	80 42
Ind'nap'lis, Ind	39 55	86 5	Wilmington, Del...	34 14	77 57
Jeffer' City, Mo	38 36 N	92 8	Worcester, Mass...	42 16	71 48
Key West, Fla,	24 33	81 47	Yorktown, Va.....	37 13	76 34

A difference of 15 degrees of Longitude equals a difference of one hour of time. The degrees of Longitude between two cities, multiplied by 4, equals, in minutes, the difference of time.

For a difference of	There is a difference of	For a difference of	There is a difference of
15° in Long.	1 hr. in Time.	1° " "	4 min." "
15′ " "	1 min." "	1′ " "	4 sec. " "
15″ " "	1 sec. " "	1″ " "	1-15 sec. in time

EUROPE, ASIA, AFRICA, AND THE OCEANS.

Place.	Lat.	Long.	Place.	Lat.	Long.
	° ′	° ′		° ′	° ′
Antwerp	51 13 N	4 24 E	Leghorn.........	43 32	10 18 E
Alexandria	31 12	29 53	Leipsic	51 20	12 22
Archangel........	64 32	40 33	Lisbon...........	38 42	9 9 W
Athens	37 58	23 44	Moscow	55 40	35 33 E
Aleppo	36 11	37 10	Malta	35 54	14 30
Algiers	36 47	3 4	Messina	38 12	15 35
Amsterdam......	52 22	4 53	Mocha...........	13 20	43 12
Borneo	5	115	Muscat	23 37	58 35
Botany Bay......	34 2	151 13	Marseilles.......	43 18	5 22
Barcelona	41 23	2 11	Manilla..........	14 36	121 2
Bombay..........	18 56	72 54	Madras	14 4	80 16
Bremen	53 5	8 49	Madrid...........	40 25	3 42 W
Berlin	52 30	13 24	Malaga	36 43	4 26
Brussels.........	50 51	4 22	New Zealand....	34 24 S	173 1 E
Cape Clear	51 26	9 29 W	New Hebrides...	15 28	167 7
Calais............	50 58	1 51 E	Niphon	34 36 N	138 51
Constantinople ..	41 1	28 59	Naples	40 50	14 16
Canton...........	23 7	113 14	Odessa	46 28	30 44
Cronstadt	59 59	29 47	Pekin............	39 54	116 28
Copenhagen	55 41	12 34	Palermo.........	38 8	13 22
Cape of G. Hope.	33 56 S	18 29	Paris	48 50	2 20
Calcutta..........	22 34 N	88 20 E	Rome............	41 54	12 27
Corinth..........	37 54	22 52	Rotterdam.......	51 54	4 29
Cairo	30 3	31 18	Smyrna..........	38 26	27 7
Ceylon...........	9 49	80 23	Singapore	1 17	103 50
Dublin...........	53 23	6 20 W	Siam.............	14 55	100
Dover............	51 8	1 19 E	Sierra Leone	8 30	13 18 W
Edinburgh.......	55 57	3 12 W	St. Helena	15 55 S	5 45
Feejee Group....	17 41 S	178 53 E	Suez	29 59 N	32 34 E
Florence.........	43 46 N	11 16	Stockholm.......	59 21	18 6
GREENWICH	51 29	....	St. Petersburgh .	59 56	30 19
Geneva	46 12	6 9	Toulon	43 07	5 22
Glasgow.........	55 52	4 16 W	Tripoli	34 54	13 11
Gibraltar	36 7	5 22	Tunis............	36 47	10 6
Genoa............	44 24	8 53 E	Tangier..........	35 47	5 54
Honolulu	21 19	157 52 W	Venice	45 50	12 26
Hamburg	53 33	9 58 E	Vienna...........	48 13	16 23
Havre............	49 29	6 W	Warsaw	52 13	21 2
Jerusalem	31 48 N	37 20 E	Zanzibar	6 28 S	39 33
Liverpool	53 25	3 W			

MEASURE OF CIRCLES, OR ANGLES.

The UNIT is the *degree*, which is 1-360 part of the circumference of any circle.

60 Seconds (″)	= 1 Minute.	′
60 Minutes	= 1 Degree.	°
30 Degrees	= 1 Sign.	S
12 Signs, or 360°	= 1 Circle.	C

AVOIRDUPOIS WEIGHT.

The price per lb. being given, to find the cost of any quantity.

Reduce the quantity to lbs. and multiply by the price per lb. To reduce cwts., &c., to lbs., two places to the right of the number of cwts. put the number of lbs. contained in the qrs. and lbs., and add the product of the cwts. multiplied by 12.

Find the cost of 93 cwts. 3 qrs. 11 lbs. @ 8d. per lb.

$$\begin{array}{r} 9395 \\ \underline{1116} \\ 10511 \text{ lbs.} \end{array} \qquad \begin{array}{l} 10511 \times 8 = 84088 \text{ pence} = £350\ 7s.\ 4d. \\ \text{or } \dfrac{1051{\cdot}1}{3} = 350{\cdot}366 = £350\ 7s.\ 4d. \end{array}$$

The price per cwt. being given, to find the cost.

Find the cost at 1s. per cwt., and multiply by the number of shillings and parts of a shilling in the price.

To find the cost of any quantity at 1*s. per cwt.*

Regard the cwts. as shillings, and 3 times the qrs. + $\frac{1}{9}$ the lbs. as pence. lbs. $\times$ ·009=decimals of 1 cwt., nearly.

17cwt. 3qrs. 20lbs. @ 1s. per cwt. = 17s. $11\frac{1}{4}$d.

17cwt. 3qrs. 20lbs. @ 3s. 4d.=17s. $11\frac{1}{4}$d. $\times$ $3\frac{1}{3}$=£2 19s. $9\frac{1}{2}$d.

The price per ton being given, to find the cost.

Multiply the cost at £1 per ton by the number of £ per ton.

*To find the cost of any quantity at £*1 *per ton.*

Regard the tons as £, the cwts. as shillings, and 3 times the qrs. + $\frac{1}{9}$ the lbs. as pence; or write the quantity decimally, and regard the figures as £ and decimals of £1 sterling.

3 tons 17 cwt. 3 qrs. 10 lbs. @ £1 = £3 17s. 10d. or £3·8916.

To write cwts. qrs. and lbs. in decimals of a ton.

$\frac{1}{10}$ the number of cwts. $\div$ 2 = tons and decimals of a ton.

75 cwts. = 7·5 $\div$ 2 = 3·75 = 3 tons 3 qrs. of a ton.

1 cwt. = ·05. 1 qr. = ·0125. 2 qrs. = ·025. 3 qrs. = ·0375.

$\frac{1}{2}$ the number of lbs. $\times$ ·0009 = decimals of a ton nearly, the excess is only $\frac{1}{10000}$ of a ton for each 28 lbs.

28 lbs. = 14 $\times$ ·0009 = ·0126 $-$ $\frac{1}{10000}$ = ·0125.

3 tons 17 cwt. 3 qrs. 10 lbs. at 17s. 6d. per ton = £3 8s. $1\frac{1}{4}$d.

£3 17s. 10d. $\times$ $\frac{7}{8}$=£3 8s. $1\frac{1}{4}$d. or 3·892 $-$ $\frac{1}{8}$=3·4055=£3 8s. $1\frac{1}{4}$d.

*To find the cost of any quantity in £ and decimals of £*1.

Regard the given number as £, and multiply by the number of pounds or parts of £1 in the price of one, or multiply ·1 the number by $\frac{1}{2}$ the price of 1 in shillings.

2793 @ 3s. 4d. = 2793 $\times$ $\frac{1}{6}$ = 465·5 = £465 10s. 0d.

2793 @ 3s. 4d. = 279·3 $\times$ $1\frac{2}{3}$ = 465·5 = £465 10s. 0d.

747 tons @ 2s. = 74·7 = £74 14s. 0d.

747 tons @ 16s. = 74·7 $\times$ 8 = 597·6 = £597 12s. 0d.

£$9\frac{1}{3}$ or £9 6s. 8d. × number of pence per lb. = value of 1 ton.
£$2\frac{1}{3}$ or £2 6s. 8d. × number of $\frac{1}{4}$d. per lb. = value of 1 ton.
£$1\frac{1}{6}$ or £1 3s. 4d. × 8ths of a penny per lb. = value of 1 ton.
The price per ton in £ × $\frac{6}{7}$ = the price per lb. in 8ths of 1d.
The price per ton in £ × $\frac{3}{7}$ = the price per lb. in farthings.
The price per ton in £ × $\frac{3}{28}$ = the price per lb. in pence.
The price per ton in £ regarded as shillings = the price per cwt.
The price per cwt. in shillings × $\frac{3}{7}$ = the price of 1 lb. in farthings.
The price of 1 lb. @ 21s. per cwt. = $\frac{21 \times 3}{7}$ = 9 far. = $2\frac{1}{4}$d.
The price of 1 lb @ £3 7s. 8d. per cwt. = 67·66 × $\frac{3}{7}$ = 29 farthings = $7\frac{1}{4}$d.
Lbs. regarded as pence × $2\frac{1}{7}$ = their value at £1 per cwt.
112 lbs. × $2\frac{1}{7}$ = 240. 240d. = £1.

To find the cost of 112 *lbs.—the price of one being given.*
Multiply 9s. 4d. by the number of pence in the price of 1 lb., or, multiply 2s. 4d. by the number of farthings in the price of 1 lb., or, to the cost of 100 lbs. add 1s. for each penny in the price of one.
112 at $2\frac{1}{4}$d. = 9s. 4d. × $2\frac{1}{4}$ = 21s., or 2s. 4d. × 9 = 21s.

To find the cost of 100—*the price of one being given in pence.*
Multiply 8s. 4d. by the number of pence, or 2s. 1d. by the number of farthings in the price of one; or write the price of one and place a decimal point two places to the right.
100 @ $2\frac{1}{4}$d. each = 8s. 4d. × $2\frac{1}{4}$, or 2s. 1d. × 9 = 18s. 9d.
The price of 1 @ $2\frac{1}{4}$d = 2·25. 2·25d. × 100 = 225d. = 18s. 9d.
100 @ $7\frac{1}{4}$d. ea. = $7\frac{1}{4}$d. × 100 = 7·25 × 100 = 725d. = £3 0s. 5d.
112 lbs. or 1 cwt. @ $7\frac{1}{4}$d. = £3 0s. 5d. + 7s. 3d. = £3 7s. 8d.

The price per 100 *being given to find the cost of one.*
Remove the decimal point two places to the left.
The cost of one @ 725d. per 100 = 725 ÷ 100 = 7·25 = $7\frac{1}{4}$d.

A sack of flour weighs 280 lbs. = 20 stones.
The price of 1 stone in shillings = the price of a sack in £.
A sack of flour @ 3s. 6d. or $3\frac{1}{2}$ s. a stone = £$3\frac{1}{2}$ = £3 10s.
280 farthings = 5s. 10d., therefore 5s. 10d. × the number of farthings in the price of 1 lb. = the price of a sack; or the number of farthings in the price of 1 lb. regarded as pence × 70 = the price of a sack in pence.
Find the price of a sack at 3 pence per lb.
5s. 10d. × 12 = £3 10s. 12d. × 70 = 840d. = £3 10s.
The price of a sack in pence ÷ 70 = the price of 1 lb. in farthings, 840 ÷ 70 = 12. 12 farthings = 3d.

To find how many cwts. in any number of lbs.

Multiply the thousands by 9, to the product add the hundreds, and deduct 8 for each 1000lbs., and 12 for each 100lbs.

3696 lbs. $= \overline{3 \times 9} + 6 = 33$ cwts. $\overline{8 \times 3} + \overline{6 \times 12} = 96$.

7369 lbs. = 65 cwts. 3 qrs. 5 lbs. $\overline{7 \times 9} + 2 = 65$.

$\overline{7 \times 8} + \overline{2 \times 12} = 80$. $169 - 80 = 89$. 89 lbs. = 3 qrs. 5 lbs.

To find the cost of 1 *ounce—the price per lb. being given.*

Regard the shillings per lb. as farthings and multiply by 3. The cost of 1 oz. @ 4s. a pound $= 4 \times 3 = 12$ farthings = 3d.

Or, the price per pound in shillings $-$ $\frac{1}{4}$ itself regarded as pence = the price per ounce.

9s. per lb. = $6\frac{3}{4}$d. per oz. $\frac{1}{4}$ of $9 = 2\frac{1}{4}$. $9 - 2\frac{1}{4} = 6\frac{3}{4}$.

To find the cost of 1 *lb.—the price per ounce being given.*

Regard the farthings in the price as shillings and divide by 3. The cost of 1 lb. @ $2\frac{1}{4}$d. per oz. $= 9 \div 3 =$ 3s.

Or, the price per ounce in pence + $\frac{1}{3}$ itself regarded as shillings = the price per pound.

3d. per oz. = 4s. per pound. $\frac{1}{3}$ of $3 = 1$. $3 + 1 = 4$.
$\frac{1}{4}$ the price of 1 lb. in pence = the price of 1 oz. in farthings.
4d. per lb. = 1 farthing per oz. $\frac{4}{4} = 1$.
9d. per lb. = $2\frac{1}{4}$ farthings per oz. $\frac{9}{4} = 2\frac{1}{4}$.

To find the cost of pounds and ounces, the price per pound in pence being given.

Find the cost of the pounds and half-pounds at the given price, and add $\frac{1}{4}$ of a farthing per oz. for each penny, or 1 farthing per oz. for each 4 pence in the price per lb.

Or, find the cost for the next higher number of pounds and half-pounds, and deduct for each 4 pence in the price per lb. 1 farthing for each ounce over the actual weight.

The cost of 13 lbs. 5 ozs. of beef @ 4d. lb. = 4s. $5\frac{1}{4}$d.
13 lbs. @ 4d. = 4s. 4d. 4s. 4d. + 5 farthings = 4s. $5\frac{1}{4}$d.
Or, $13\frac{1}{2}$ lbs. @ 4d. = 4s. 6d. 4s. 6d. − 3 farthings = 4s. $5\frac{1}{4}$d.
The cost of 8 lbs. 13 ozs. @ 8d. per lb. = 5s. $10\frac{1}{2}$d.
$8\frac{1}{2}$ lbs. @ 8d. = 5s. 8d. 5s. 8d. + 5 halfpence = 5s. $10\frac{1}{2}$d.
Or, 9 lbs. @ 8d. = 6s. 6s. − 3 halfpence = 5s. $10\frac{1}{2}$d.

To find the cost of any quantity—the price of one being given.

Find the amount at 1d. or 1s. and multiply by the number of pence or shillings each.

38 lbs. of beef at $7\frac{1}{2}$d. = 3s. 2d. $\times 7\frac{1}{2} =$ £1 3s. 9d.
The cost of 40 articles at 3s. 6d. = £2 $\times 3\frac{1}{2} =$ £7.

	s.	d.		s.	d.		£	s.	d.
12 pence =	1	0	70 pence =	5	10	240 pence =	1	0	0
20 pence =	1	8	72 pence =	6	0	360 pence =	1	10	0
24 pence =	2	0	80 pence =	6	8	480 pence =	2	0	0
30 pence =	2	6	84 pence =	7	0	600 pence =	2	10	0
36 pence =	3	0	90 pence =	7	6	720 pence =	3	0	0
40 pence =	3	4	96 pence =	8	0	840 pence =	3	10	0
48 pence =	4	0	100 pence =	8	4	960 pence =	4	0	0
50 pence =	4	2	108 pence =	9	0	1080 pence =	4	10	0
60 pence =	5	0	120 pence =	10	0	1200 pence =	5	0	0

To find the cost of any number of units at one penny each.

From the given number subtract the pence representing the £; to the pounds annex the remaining shillings and pence.

747@1d. ea., 747 − 720=27, 720d.=£3, £3 + 27d.=£3 2s. 3d.

When the number of pence is several thousands, divide the hundreds by 12, and regard each unit in the quotient as £5.

4875d. = $\overline{48 \div 12} \times 5 = 20$. £20 + 75d. = £20 6s. 3d.

When the price is a number of pence each.

Find the cost at 1d. each, and multiply by the price of one.

747 @ 7d. each = £3 2s. 3d. × 7 = £21 15s. 9d.

This method of reducing pence to £ s. d. may be utilised in reckoning Interest; any number of £ sterling regarded as pence = the interest for one month at 5 per cent. per annum.

The int. on £747 for 1 month = 720 + 27d. = £3 2s. 3d.

TABLE FOR MARKING ALL GOODS BOUGHT BY THE DOZEN.

Removing the decimal point one place to the left on the cost of a dozen shows the cost of one with 20 per cent. added.

To make 20 % remove the decimal point 1 place to the left.

To make 25 %	„ and add	$\frac{1}{24}$	To make 40 %	and add	$\frac{1}{6}$
„ „ 26 %	„ „ „	$\frac{1}{20}$	„ „ 44 %	„	$\frac{1}{5}$
„ „ 28 %	„ „ „	$\frac{1}{15}$	„ „ 50 %	„	$\frac{1}{4}$
„ „ 30 %	„ „ „	$\frac{1}{12}$	„ „ 60 %	„	$\frac{1}{3}$
„ „ 32 %	„ „ „	$\frac{1}{10}$	„ „ 80 %	„	$\frac{1}{2}$
„ „ 33⅓%	„ „ „	$\frac{1}{9}$	„ „ 12½%	subtract	$\frac{1}{16}$
„ „ 35 %	„ „ „	$\frac{1}{8}$	„ „ 16⅔%	„	$\frac{1}{36}$
„ „ 37½%	„ „ „	$\frac{1}{7}$	„ „ 18¾%	„	$\frac{1}{96}$

To find the cost of dozens at a given number of pence each.

Regard the number of pence in the price of one as shillings, and multiply by the number of dozens.

4 doz. @ 2½d. each = 2s. 6d. × 4 = 10s.

The price per gross being given, to find the cost of one.

Regard the shillings per gross as farthings, and divide by 3

The cost of 1 @ 6s. per gross = 6 ÷ 3 = 2 farthings.

The price of one being given, to find the cost per gross.

Regard the farthings each as shillings, and multiply by 3

1 gross @ 3d. each = 12 × 3 = 36 shillings.

GOLD COINS their weight, fineness, and value in British and United States money, based on U. S. Mint assays, computed by C. FRUSHER HOWARD.

Country.	Denomination.	Weight.		Fineness.		Value.	
		Grains.	Ounces.	1000ths	Carats.	£ s. d.	U. S. $
Austria,	Union Crown,	171.36	0.357	900.	21.60	1,, 7,, 3½	6.6419
Belgium,	25 Francs,	121.92	0.254	899.	21.57	19,, 4½	4.7203
Bolivia,	Doubloon,	416.16	0.867	870.	20.88	3,, 4,, 1	15.5925
Brazil,	20 Milries,	276.00	0.575	917.5	22.02	2,, 4,, 10	10.9057
Chili,	Doubloon,	416.16	0.867	870.	20.88	3,, 4,, 1	15.5925
Denmark,	10 Thaler,	204.96	0.427	895.	21.48	1,,12,, 5½	7.9000
England,	Sovereign,	123.27	0.2568	916.6	22.00	1,, 0,, 0	4.8665
France,	20 Francs,	99.60	0.2075	899.	21.57	15,,10¼	3.8562
Germany,	20 Marks,	122.90	0.256	900.	21.60	19,, 6½	4.7627
Greece,	20 Drachms,	88.80	0.185	900.	21.60	14,, 1¾	3.4419
India,	Mohur,	179.52	0.374	916.	22.00	1,, 9,, 1	7.0818
Italy,	20 Lire,	99.36	0.207	898.	21.55	15,, 9¼	3.8426
Japan,	5 Yen,	128.30	0.267	900.	21.60	1,, 0,, 5	4.9674
Mexico,	Doubloon,	416.16	0.867	870.5	20.89	3,, 4,, 1½	15.6105
"	20 Pesos,	518.88	1.081	873.	20.95	4,, 0,, 2	19.5083
Netherl'ds.	10 Guilders,	103.72	0.216	899.	21.57	16,, 6	4.014
Peru,	Doubloon,	416.16	0.867	868.	20.83	3,, 3,,11¼	15.5567
"	20 Soles,	496.80	1.035	898.	21.55	3,,18,,11½	19.2130
Portugal,	Gold Crown,	147.84	0.308	912.	21.88	1, 3,,10½	5.8066
Rome,	2½ Scudi,	67.20	0.140	900.	21.60	10,, 8	2.6047
Russia,	5 Roubles,	100.80	0.210	916.	22.00	16,, 4	3.9764
Spain,	100 Reales,	128.64	0.268	896.	21.50	1,, 0,, 5	4.9639
Sweden,	Ducat,	53.28	0.111	975.	23.40	9,, 2	2.2372
Turkey,	100 Piasters,	110.88	0.231	915.	21.96	17,,11½	4.3693
United States.	20 Dollars,	516.00	1.075	900.	21.60	4,, 2,, 2½	20.0000
	One Dollar.	25.80	.05375	900.	21.60	.2054838	1.0000

The market value of any silver coin, in pence, = the number of ounces of pure silver in it ÷ 98 × the market price in pence.

Ounces of Gold × the fineness × 20·6718 = U.S. Dollars.

Ounces of Gold × the fineness × 4·248 = £ Sterling.

Table of various Silver Coins, showing their weight, fineness and quota of pure silver, computed from U. S. Mint assays, by C. FRUSHER HOWARD.

Country.	Denomination.	Fineness.	Weight.		Pure Silver.	
			Ounces.	Grains.	Grains.	Cunces.
Austria,	New Florin,	.900	0.397	190.56	171.504	.357300
"	" Dollar,	.900	0.596	286.08	257.472	.536400
Belgium,	5 Francs,	.897	0.803	385.44	345.739	.72029
Bolivia,	New Dollar,	.9035	0.643	308.64	278.856	.580950
Brazil,	Double Milries.	.9185	0.820	393.60	361.521	.753170
Canada,	20 Cents,	.925	0.150	72.00	66.600	.138888
Cen. America.	Dollar,	.850	0.866	415.68	353.328	.736100
Chili,	New Dollar,	.9005	0.801	384.48	346.224	.721300
China, Hong K.	English Dollar,	.901	0.866	415.68	374.527	.780266
Denmark,	Two Rigsdaler,	.877	0.927	444.96	390.230	.812979
England,	New Shilling,	.925	0.1818	87.27	80.727	.168181
France,	1 Franc,	.900	0.16075	77.68	69.444	.144675
Germany,	Mark,	.900	0.1785	85.70	77.112	.160687
Greece,	5 Drachms,	.900	0.719	345.12	310.608	.647100
East Indies,	Rupee,	.9166	0.375	180.00	165.00	.34375
Japan,	New Dollar,	.900	0.875	420.00	378.000	.787500
Mexico,	" "	.903	0.8675	416.40	376.009	.783352
Naples,	Scudo,	.830	0.844	405.12	336.249	.700518
Holland,	$2\frac{1}{2}$ Guildeis,	.944	0.804	385.92	364.308	.758975
Norway,	Specie Daler,	.877	0.927	444.96	390.229	.812977
Peru,	Dollar 1858,	.909	0.766	367.68	334.221	.696294
Rome,	Scudo,	.900	0.864	414.72	373.248	.777600
Russia,	Rouble,	.875	0.667	320.16	280.140	.583625
Spain,	New Pistareen,	.899	0.166	79.68	71.632	.149233
Sweden,	Rix Daler,	.750	1.092	524.16	393.120	.819000
Turkey,	20 Piasters,	.830	0.770	369.60	306.768	.639100
Tuscany,	Florin,	.925	0.220	105.60	97.680	.203500
United States.	Dollar	.900	0.8594	412.50	371.25	.7734375
" "	Trade "	.900	0.875	420.00	378.00	.787500

COAL MINERS' RECKONINGS.

In reckoning the quantity of coal raised from the pit the usage differs in different districts.

TYNE COLLIERIES.

In the Tyne Collieries 20 Tubs=1 score, 8 cwt. 1 qr.=1 Tub.

To find the value of any number of Tubs, the price per score being given, 20 Tubs to the score.

RULE.—*Multiply one-tenth,* ·1 *the price per score in pence, by half the number of Tubs.*

10 Tubs @ 6s. per score $= 7{\cdot}2 \times 5 = 36$d. $=$ 3s.

7 Tubs @ 6s. per score $= 7{\cdot}2 \times 3\frac{1}{2} = 25{\cdot}2$d. $=$ 2s. $1\frac{1}{4}$d.

Find the value of 6 score and 8 Tubs @ 6s. per score.

6 score=36s. $7{\cdot}2 \times 4 = 28{\cdot}8$d. $=$ 2s. $4\frac{3}{4}$d. 36s. + 2s. $4\frac{3}{4}$d $=$ £1 18s. $1\frac{3}{4}$d.

Find the value of 5 score and 9 Tubs @ 7s. per score.

5 score $=$ 35s. $8{\cdot}4$d. $\times 4\frac{1}{2} = 37{\cdot}8$d. 35s. + 37.8 $=$ £1 18s. $1\frac{3}{4}$d.

DURHAM COLLIERIES.

In the Durham Collieries 21 Tubs $=$ 1 score.

To find the value of any number of Tubs 21 to the score.

Find the value, as before, and deduct $\frac{1}{4}$d. for each 5 *pence.*

10 Tubs @ 6s. per score $= \overline{7{\cdot}2\text{d.} \times 5} - 7$ farthings $= 34\frac{1}{4}$d. $=$ 2s. $10\frac{1}{4}$d.

Find the value of 18 Tubs @ 7s. per score.

$8{\cdot}4 \times 9 = 75{\cdot}6$d. $75\frac{1}{2}$d. $-$ 15 farthings $= 71\frac{3}{4}$d. $=$ 5s. $11\frac{3}{4}$d.

Find the value of 9 Tubs @ 7s. per score.

$8{\cdot}4$d. $\times 4\frac{1}{2} = 37{\cdot}8$d. $37\frac{3}{4}$d. $-$ 7 farthings $=$ 3s.

The Royalty is paid on each 6 cwts.

RECKONINGS MADE AT A GIVEN RATE PER CENT.

One per cent. means 1 for each 100, and is shown by writing the principal in figures, and then placing a decimal point between the second and third figures from the right, thus: 1 per cent. of $100 = 1{\cdot}00 = 1$. 1 per cent. of $750 = 7{\cdot}50 = 7\frac{1}{2}$. 1 % of $375 = 3{\cdot}75 = 3\frac{3}{4}$. 1 % of $780 = 7{\cdot}8 = 7\frac{8}{10}$.

To find the percentage on any quantity at any Rate per cent. Remove the decimal point two places left and multiply by the Rate.

4 % of $750 = 7{\cdot}5 \times 4 = 30{\cdot}0 = 30$.

2 % of £487 10s. 0d. $= 4{\cdot}875 \times 2 = 9{\cdot}75 =$ £9 15s. 0d.

$3\frac{1}{4}$ % of £275 7s. 6d. $= 2{\cdot}75375 \times 3\frac{1}{4} = 8{\cdot}95 =$ £8 19s. 0d.

See pages 44, 59, &c.

THE METRIC SYSTEM of Weights and Measures is based upon the decimal scale; its paramount simplicity insures its early adoption by all civilised nations.

The **Meter** is the base of the system, and $=39.37079$ in. Meters $+\frac{1}{12}+\frac{1}{8}$ of $\frac{1}{12}$=yds.

The **Are** (*air*) is the unit of surface, the **Stere** (*stair*) is the unit of volume, the **Litre** (*leeter*) is the unit of capacity, the **Gram** is the unit of weight; these constitute the primary units of the system.

The **Multiple Units,** or higher denominations, are named by prefixing to the name of the primary units the *Greek* numerals *Deka* (10), *Hecto* (100), *Kilo* (1,000), and *Myra* (10,000).

The submultiple units, or lower denominations, are named by prefixing to the names of the lower denominations the *Latin* numerals, *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), *Milli* ($\frac{1}{1000}$).

The **Name** of a unit indicates whether it is *greater* or *less* than the standard units, and also how many times.

MEASURES OF EXTENSION.—The **Meter** is the unit of length, and=39.37079 inches, and is used in measuring cloths and short distances. Yds. — $\frac{1}{12}$=meters, nearly.

The **Kilometer** is commonly used for measuring long distances, and is about five-eights of an English mile. Kilometers × .621= miles, nearly; miles + .6=kilometers, nearly.

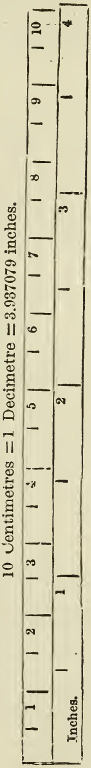


TABLE.

Metric Denominations.			English or U.S values.
		1 Millimeter	= .03937079 in.
10 Millimeters,	*mm.*	=1 Centimeter	= .3937079 ,,
10 Centimeters,	*cm.*	=1 Decimeter	= 3.937079 ,,
10 Decimeters,	*dm.*	=1 **Meter**	=39.37079 ,,
10 METERS,	*M.*	=1 Dekameter	=32.808992 ft.
10 Decameters,	*Dm.*	=1 Hectometer	=19.88423 rd.
10 Hectometers,	*Hm.*	=1 Kilometer	= .6213824 mi
10 Kilometers,	*Km.*	=1 Myrameter	= 6.213824 ,,

The ***Are*** is the *unit of land measure,* and is a square whose side is 10 meters, equal to a *square dekameter,* or 119.6 sq. yards.

Hectares × 2.47=acres, nearly; acres × .405=Hectares, nearly.

TABLE.

1 Centiare,	*ca.*	=(1 Sq. Meter)	=	1.196034 sq. yd.
100 Centaires,	*ca.*	= 1 *Are*	=	119.6034 sq. yd.
100 ARES,	*A.*	= 1 Hectare (*Ha.*)	=	2.47114 acres.

The ***Square Meter*** is the *unit* for measuring ordinary surfaces; as floorings, ceilings, &c. Square ft. × ·0929=Meters, nearly.
Square Meters × 10·7643=feet, square yards × ·836=Meters.

TABLE.

100 Sq. Millimeter,	*sq. mm.*=1 Sq. Centimeter	= .155+	sq. in.
100 Sq. Centimeters,	*sq. cm.*=1 Sq. Decimeter,	=15.5+	sq. in.
100 Sq. Decimeters,	*sq. dm.*=1 ***Sq. Meter*** (*Sq. M.*)=	1.196+	sq. yd.

The ***Stere*** is the *unit* of wood or solid measure, and is equal to a *cubic meter,* or .2759 cord.

TABLE.

		1 Dicistere	= 3.531+ cu. ft.
10 Dicisteres,	*dst.*	=1 ***Stere***	=35.316+ cu. ft.
10 STERES,	*St.*	=1 Dekastere (*DSt.*)	=13.079+ cu. yd.

The ***Cubic Meter*** is the *unit* for measuring ordinary solids; as excavations, embankments, etc.=35.3165818 cu. feet.

TABLE.

1000 Cu. Millimeters,	*cu. mm.* =1 Cu. Centimeter=	.061+	cu. in.
1000 Cu. Centimeters,	*cu. cm.* =1 Cu. Decimeter	=61.026+	cu. in.
1000 Cu. Decimeters,	*cu. dm.* =1 ***Cu. Meter***	=35.316+	cu. ft.

MEASURES OF CAPACITY.

The ***Liter*** is the *unit of capacity,* both of Liquid and Dry Measures, and is a vessel whose volume is equal to a cube whose edge is *one-tenth* of a *meter,* equal to 1.05673 Liquid Measure. and .9081 quart Dry Measure. U.S. or 1.76077 imp. pints=61.027048 cu. inches.

Imperial Pints × .568 = Liters; Liters × 1.76 = Imperial Pints, nearly.

TABLE.

10 Milliliters,	*ml* =1 Centiliter.	10 Dekaliters,	*Dl.*=1 Hectoliter.
10 Centiliters,	*cl.* =1 Deciliter.	10 Hectoliters,	*Hl.*=1 Kiloliter.(Stere)
10 Deciliters,	*dl.* =1 ***Liter.***	10 Kiloliters,	*Kl.*=1 Myrialiter.
10 LITERS,	*L.* =1 Dekaliter.		(*Ml.*)

The ***Hectoliter*** is the *unit* in measuring liquids, grain, fruit and roots in large quantities=22·00965 imperial gallons.

Equivalents in U. S. and Imperial Measures.

Metric Denominat'ns.	Cubic Measure.	Dry Measure.	Wine Measure.	Imp. Measure.
1 Myrialiter	=10 Cubic Meters	=283.7 bu.	=2641. gal.	=2200.96 gal.
1 Kiloliter	= 1 Cubic Meter	=28.37 bu.	=264.17 gal.	= 220.096 gal.
1 Hectoliter	$=\frac{1}{10}$ Cubic Meter	=2.837 bu.	=26.417 gal.	= 22.0096 gal.
1 Dekaliter	=10 Cu. Decimet'r	=9.08 qts.	=2.6417 gal.	= 2.20096 gal.
1 *Liter*	= 1 Cu. Decimet'r	=.908 quart	=1.0567 qt.	= 7.0430 gills
1 Deciliter	$=\frac{1}{10}$ Cu. Decimet'r	=6.102 c. in	=.845 gill	= .7043 gill.
1 Centiliter	=10 Cu. C'ntim't'r	=.6102 cu. in	=.338 fl'd oz.	= .0704 gill.
1 Milliliter	= 1 Cu. C'ntim't'r	=.061 cu. in	=.27 fl'd dr.	= .0070 gill.

Measures of Weight.

The *Gram* is the *unit of weight*, and equal to the weight of a cube of distilled water, the edge of which is *one hundredth* of a *meter*, equal to 15.4323487 Troy grains.

The ***Kilogram,*** or ***Kilo,*** is the *unit* of common weight in trade, Kilos × 2.2 or $2\frac{1}{5}$=lbs. avoirdupois, nearly.

The ***Tonneau*** is used for weighing very heavy articles, and is about 204 lbs. more than the U.S. ton; about 35 lbs. less than the long ton ·9842 long ton = 1 Tonneau; Tonneau ÷ 60 — the quotient = long tons, nearly.

Table.

10 Milligrams,	*mg.*	=1 Centigram	=	.15432+ grains
10 Centigrams	*cg.*	=1 Decigram	=	1.54323+ " "
10 Decigrams,	*dg.*	=1 ***Gram***	=	15.43234+ " "
10 Grams,	*G.*	=1 Dekagram	=	.35273+ oz. Avoir.
10 Dekagrams,	*Dg.*	=1 Hectogram	=	3.52739+ " "
10 Hectograms,	*Hg.*	=1 Kilogram, or ***Kilo,***	=	2.20462+ lb. "
10 Kilograms,	*Kg.*	=1 Myriagram	=	22.04621+ " "
10 Myriagrams, or 100 Kilograms	*Mg.*	=1 Quintal	=	220.46212+ " "
10 Quintals, or 1000 Kilos,		=1 Tonueau, or ***Ton,***	=	2204.62125 " "

Comparison of the Common and Metric Systems.

1 Inch,	=	2.54 Centimeters
1 Foot,	=	30.48 Centimeters
1 Yard,	=	.9144 Meters
1 Rod,	=	5.029 Meters
1 Mile,	=	1.6093 Kilometers
1 Sq. in.	=	6.4528 Sq. Centim't'rs
1 Sq. ft.,	=	929 Sq. Centimeters
1 " yard,	=	8.361 Sq. Meters
1 " rod	=	25.29 Centairs
1 Acre,	=	40.47 Ares.
1 Sq. mile,	=	259 Hectares
1 Cu. in.	=	16.39 Cu. Centim't'rs
1 " ft.,	=	28320 " "
1 " yd.,	=	.7646 " Meters.
1 Cord,	=	3.625 Steres
1 Fl. ounce,	=	2.958 Centiliters
1 Gallon, U.S.	=	3.786 Liters
1 Bushel, U.S.	=	.3524 Hectoliter
1 Troy gr.	=	64.8 Milligrams
1 " lb.	=	.373 Kilo
1 Av. lb.	=	.4536 "
1 Ton, U.S.	=	.907 Tonneau

Méthode de Calcul pour l'espace de Trente Siècles.

Règle. Des deux derniers chiffres de l'an, rejetez tous les sept, tout en retenant le restant; divisez les deux derniers chiffres de l'an par *quatre*, retenant le quotient, sans tenir compte du restant, s'il y en a ; puis prenez le jour du mois, ensuite le chiffre donné pour le mois, et finalement celui pour le siècle. Ayez toujours soin de rejeter les *sept* où il y en a.

Le chiffre 1 (un) restant représente le premier; 2, le second; &c., et O (zéro) le dernier jour de la semaine.

TABLE DES CHIFFRES POUR LES MOIS.

1, Septembre et Déc. 3, Jan. et Oct. 5, Août.
2, Avril et Juillet. 4, Mai. 6, Fév., Mars, Nov. 0, Juin.

Nota. Dans l'année bissextile le chiffre pour Janvier est 2, et celui pour Février 5.

TABLE DES CHIFFRES POUR LES SIÈCLES.

1, est le chiffre pour les 2ème, 9ème, et 16ème, siècles.
2, " " " " " 1er, 8ème, 15ème, 18ème, 22ème, 26ème, 30ème, [siècles.
3, " " " " " 7ème, 14ème siècles.
4, " " " " " 6ème, 13ème, 17ème, 21ème, 25ème, 29ème, [siècles.
5, " " " " " 5ème, 12ème, 20ème, 24ème, 28ème, siècles.
6, " " " " " 4ème, 11ème siècles.
0, " " " " " 3ème, 10ème, 19ème, 23ème, 27ème, siècles.

Exemple. Quel fut le jour de la semaine au 31 Août, 1873 ? Réponse, Dimanche.

Procédé—

Deux derniers chiffres de l'an,	73—70=3
Quotient de 73 divisé par *quatre*,	18+3—21=0
Jour du mois,	31—28=3
Chiffre pour le mois,	5+3—7=1

Après avoir rejeté tous les *sept* il reste le chiffre 1; ce fut donc, le premier jour de la semaine, Dimanche.

N. B. Les siècles pairs non-divisibles par le chiffre 400 ne sont pas des années bissextiles.

Methode zu sagen den Tag in der Woche nach jedem Datum von Christi Geburt dreitausend Jahr.

Methode. Streich die Sieben aus von den beiden letzten Nummern auf das Jahr, der Minuent von den beiden letzten Nummern im Jahre, dividirt bei vier—gebrauche nicht den Rest den Datum auf den Monat, und die Zahlen auf das Jahr. Was überbleibt ist der Tag in der Woche, der erste Sonntag, der zweite Montag u. s. w.

Die Zahlen für die Monate.

1 vor Sept. u. Decbr.	3 vor Jan. u. Oct.	5 vor August.	0 vor Juni.
2 vor April und Juli.	4 vor Mai.	6 vor Feb., März, Nov.	

Der Datum im Januar und Februar ist eins weniger im Schaltjahr.

Datum in den Jahren.

1, ist die Zahlen vor das	2te, 9te und 16te Jahrhundert.
2, „ „ „ „ „	1te, 8te, 15te, 18te, 22te, 26te und 30te Jahrhundert.
3, „ „ „ „ „	7te, 14te Jahrhundert.
4, „ „ „ „ „	6te, 13te, 17te, 21te, 25te, 29te Jahrhundert.
5, „ „ „ „ „	5te, 12te, 20te, 24te, und 28te Jahrhundert.
6, „ „ „ „ „	4te und 11 Jahrhundert.
0, „ „ „ „ „	3te, 10te, 19te, 23te und 27te Jahrhundert.

Exempel. Welcher Tag in der Woche war der 31. August, 1873? Antwort, Sonntag.

Die letzten beiden Zahlen im Jahre,	$73 - 70 = 3$
Minuent auf do. $\div$ bei vier,	$18 + 3 - 21 = 0$
Datum im Monat,	$31 - 28 = 3$
Zahlen auf den Monat,	$5 + 3 - 7 = 1$

Der Rest 1 zeigt Euch den ersten Tag in der Woche, welcher ist Sonntag.

To find the figure for any Century from the 1st to the 16th, multiply the figures expressing the hundreds in the given year by 6, add 2, and divide by 7; the remainder is the figure for the Century.

To find the figure for the 17th and succeeding Centuries, subtract 16 from the number of hundreds in the given year, multiply by $5\frac{1}{4}$, to the product—less the fraction—add 4, and divide by 7; the remainder is the figure for the Century.

NOTE Between the Julian and the Gregorian Calendars there was a difference of ten days in 1583 and of eleven days in 1753. At the present time the difference is twelve days. The latter came into use in Catholic countries in 1583 and in England in 1753.

Howard's New Style Calendar for Thirty Centuries.

RULE. Cast all the sevens out of the last two figures of the year; add the remainder to the quotient* of the last two figures of the year, divided by four; take this sum with the day of the month, the figure for the month, and the figure for the century, dropping all the sevens as they occur, one remainder will be the first day of the week, Sunday; 2, the second, &c.; 0, last day of the week, Saturday.

* Disregard the fraction, if any, in the quotient.

TABLE OF FIGURES FOR THE MONTHS.

1, Sept. and Dec.	3, Jan. and Oct.	5, August.	0, June.
2, April and July.	4, May.	6, Feb., March, Nov.	

NOTE. The figure for January is 2, and February 5 in leap year; the years divisible by 4, exactly, are leap years.

The even centuries not divisible by 400 are not leap years.

TABLE OF FIGURES FOR THE CENTURIES.*

1, is the figure for the 2d, 9th, and 16th centuries.
2, " " " " " 1st, 8th, 15th, 18th, 22d, 26th, 30th centuries.
3, " " " " " 7th, 14th centuries.
4, " " " " " 6th, 13th, 17th, 21st, 25th, 29th centuries.
5, " " " " " 5th, 12th, 20th, 24th, 28 centuries.
6, " " " " " 4th, 11th centuries.
0, " " " " " 3d, 10th, 19th, 23th. 27th centuries.*

EXAMPLE. What day of the week was the 31st August, 1873? Sunday, Ans.

Process

Last two figures of the year,	73 — 70 = 3
Quotient of 73 ÷ by four,	18 + 3 — 21 = 0
Day of month,	31 — 28 = 3
Figure for the month,	5 + 3 — 7 = 1

*Pay no attention to the figure for this, the 19th century, as it is 0; for the last century, add 2; for the coming century, add 5,

After casting out the sevens the remainder is 1: hence it was on the first day of the week, Sunday.

The difference of the figure for the month, for any given year, and 7 is the date of the first Saturday in the month, thus the 1st Saturday in August, 1873, was the 2nd; 7 – 5 = 2.

HOWARD'S

Tables of Standard Weights and Measures.

A Standard Measure is a fixed unit established by law, by which quantity, as extent, dimension, capacity or value is measured.

The English Standard units are the YARD, the IMPERIAL GALLON, the OUNCE TROY, the POUND AVOIRDUPOIS, and the GOLD SOVEREIGN.

The U. S. Standard units are the YARD, the GALLON, the BUSHEL, the TROY POUND, and the GOLD DOLLAR.

The Standard unit of weight must be of definite dimensions, and of definite gravity, of some substance, a certain volume of which, under certain conditions, will always have a certain weight.

One cubic inch of pure water weighed in vacuo, thermometer 62° Fahrenheit, Barometer 30°= 252.458 grains.

480 grains = 1 Troy ounce, 7000 grains = 1 Pound avoirdupois.

In the Treasury at Washington is a brass scale which, at a temperature of 62° Fahrenheit, is 82 inches long; all U.S. weights and measures are referred to this unit.

LONG MEASURE.

IN.	FT.	YD.	RD.	FUR		
12	1	......	....	....	..1	Foot.
36	3	1	....	....	..1	Yard.
198	16½	5½	.1	....	..1	Rod.
7920	660	220	40	..1	..1	Furl'ng
63360	5280	1760	320	..8	..1	Mile.

SURVEYORS' LONG MEASURE.

IN.	L.	RD.	C.		
7.92	..1	....	...	1	Link.
198	25	..1	...	1	Rod.
792	100	..4	.1	1	Ch'n.
63360	8000	320	80	1	Mile.

The Geographical Mile equals 1.15 Statute Miles,
2240 yards=1 Irish Mile; 11 Irish Miles=14 English Miles.
Irish Miles $+\frac{3}{11}$ = English Miles. English Miles $-\frac{3}{14}$ = Irish Miles.

COMPARISON OF STANDARD MEASURES OF DISTANCES.

Country.		U. S. Mile.
Austria, ...1 Mile,		= 4.93
China, ...1 Li,		= .35
East Indies, 1 Coss,		= 1.14
Egypt, ...1 Mili,		= 1.15
England, ...1 Mile,		= 1.00
France, ...1 Kilomet'r.		= .62
Japan, ...1 Ri,		=2.562
Mexico, ...1 Silio,		= 6.76
Persia, ...1 Farsang.		= 4.17
Portugal,.....1 Milha,		= 1.28
Prussia, ...1 Meile,		= 4.93
Russia, ...1 Verst.		= .66
Spain, ...1 League,		= 4.15
Sweden, ...1 Mil,		= 6.64
Switzerland, 1 Lieue,		= 2.98
Turkey, ...1 Berri,		= 1.04

For measuring Land, Boards, Painting, Paving, Plastering, etc.

SQ. INCH.	SQ. FOOT.	SQ. YARD.	SQ. RD.	SQ. R.	SQ. A.		
144	1	....	....	.. ..	......	1	SQ. FT.
1296	9	1	....	......	......	1	YARD.
39204	272¼	30¼	1	......	......	1	ROD.
1568160	10890	1210	40	...1	......	1	ROOD.
6272640	43560	4840	160	4	...1	1	ACRE.
4014489600	27878400	3097600	102400	2560	640	...1	MILE.

Square yds. $\div 10 \times 4 \times 11 \times 11 =$ Acres.

Square yds. $\times$ .00020661 = Acres, nearly.

In measuring Roofing, Paving, etc., 100 square feet = one square.

One thousand shingles, averaging 4 inches wide, and laid 5 inches to the weather, are estimated to be a square.

One mile square=1 section=640 acres. 36 square miles (6 miles square)=1 township.

The sections are all numbered 1 to 36, commencing at the north-east corner, thus:

6	5	4	3	2	NW | NE SW | SE
7	8	9	10	11	12
18	17	16*	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

The sections are all divided into quarters, which are named by the cardinal points, as in section 1. The quarters are divided in the same way. The description of a forty-acre lot would read: The south half of the west half of the south-west quarter of section 1 in township 24, north of range 7 west, or as the case might be; and sometimes will fall short, and sometimes overrun the number of acres it is supposed to contain.

*Reserve for school purposes.

Gunter's Chain is a unit of measure, and is four Rods, or 66 feet long; it consists of 100 links. It is also common to use a chain, or measuring tape, 100 feet long, each foot divided into tenths.

The Rod is sometimes called a Pole or Perch.

1 Irish acre contains 7840 square yards, 3000 yards more than the English acre. Irish acres $\times$ 196 $\div$ 121 or $\times$ 16·2 = English acres. English acres $\times$ 121 $\div$ 196 or $\times$ ·617347 = Irish acres.

In the Pacific Coast States and Territories the divisions of Land are frequently expressed by the old Mexican Measurements:

A fifty Vara lot is 137½ feet square. 1 Vara=33-36=11-12 of a yard.

Varas $\times$ 11 $\div$ 12=yards. Yards $\times$ 12 $\div$ 11=Varas.

For measuring timber, stone, boxes, packages, capacity of rooms, etc.

CU. IN.	CU. FT.	CU. YD.	CD. FT	CD.	PCH.		
1728	1	....	......	......	......	1	Cubic Foot.
46656	27	1	......	......	......	1	Cubic Yard.
27648	16	16-27	1	......	......	1	Cord Foot.
221184	128	4 20-27	8	1	......	1	Cord of Wood.
42768	24¾	....	......	......	1	1	Perch of Stone.
69120	40	....	......	......	......	1	U.S.Ton,Ship Carge

One ton of square timber = 50 cubic feet.

The English shipping ton = 42 cu. ft. The Register ton = 100 cu. ft.

A cord of wood is a pile 4 ft. high, 4 ft. wide, and 8 ft. long.

A cord foot is one foot in length of such a pile.

A cubic yard of common earth is called a load.

In Board measure all boards are assumed to be 1 inch thick.

A board foot is 1 ft. long, 1 ft. wide and 1 in. thick, hence 12 board feet make 1 cubic foot.

Board feet are changed to cubic feet by dividing by 12.

Cubic feet are changed to Board feet by multiplying by 12.

Masonry is estimated by the CUBIC FOOT and PERCH; also by the SQUARE FOOT and SQUARE YARD.

CUBIC FEET × 4 ÷ 99 = PERCHES.

A fathom of lathwood = 6 × 6 × 6 = 216 cubic feet.

The St. Petersburg Standard = 5 × 3 × 11 = 165 cubic feet.

Cubic feet × ·00606 = St. Petersburgh Standard.

In board and lumber measure, estimates are made on 1 inch in thickness; one-fourth the price is added for every ¼ inch in thickness over one inch.

MISCELLANEOUS WEIGHTS AND MEASURES.

12 Units,............. 1 Dozen.
12 Dozen,............. 1 Gross.
12 Gross,....... 1 Great Gross.
20 Things,............ 1 Score.
196 lbs...... 1 Barrel of Flour.
200 " ..1 Bbl. Beef, Pork, Fish.
56 " ...... 1 Firkin of Butter.
14 " .... 1 Stone, Avoir.
28 "........... 1 Quarter, "
21½ Stones,..... 1 Pig of Iron.

8 Pigs ....1 Fother.
28 lbs. of Wool........1 Tod.
2 Weys (364 lb.) 1 Sack of Wool.
12 Sacks (4368 lbs.) ..1 Last.
3 Inches ....1 Palm.
4 " .... 1 Hand.
9 " ....1 Span.
3 ft......... 1 common pace.
6 " ....1 Fathom.
3 miles............1 League.

TROY WEIGHT.

For Gold, Silver, Jewels, etc.

Gr.	Pwt.	Oz.		
24	...1	....	..1	*Pennyweight*
480	20	..1	..1	*Ounce.*
5760	240	12	..1	*Pound.*

AVOIRDUPOIS WEIGHT.

For Groceries, Provisions, etc.

Gr.	Oz.	Lb.		
437½	1	....	..1	*Ounce*
7000	16	1	..1	*Pound.*
14000000	32000	2000	..1	*Ton.*

The Standard units are the Ounce Troy and the Pound Avoirdupois.
The Long Ton = 2240 lbs. 1 cwt. = 112 lbs.
The Short Ton=2000 lbs. Long Tons×1.12=Short Tons.
To compare Troy weights with Avoirdupois, reduce both to grains.
Pounds Avoirdupois × 100×7÷48= ounces Troy.

Troy ounces ×.06 6-7= Pounds avoirdupois; that is, ounces multiplied by .06+1-7 of the product.

28 lbs.=1 qr. 4 qrs =1 cwt. 20 cwts.=1 long ton.

APOTHECARIES' WEIGHT.

GRS.	SC. ℈	DR. ʒ	OZ. ℥		
20	..1	....	....	..1	SCRUPLE.
60	3	..1	....	..1	DRAM.
480	24	8	..1	..1	OUNCE.
5760	288	96	12	..1	POUND.

APOTHECARIES' MEASURE.

60 Minims = 1 Fluid Drachm.

8 Fl. Drms = 1 Fluid Ounce.

16 Fl. Ozs. = 1 Pint.

8 Pints = 1 Gallon.

Used in compounding liquid medicines.

The grain, ounce and pound are the same as Troy Weight.
Drugs are bought and sold in quantities by Avoirdupois Weight.
1 Teaspoon = 45 Drops. 1 Tablespoon = ½ Fluid Ounce.

COMPARISON OF LIQUID MEASURES.

Country.		U. S. Gals.
England, ...1	Gallon,	= 1.2
France, ...1	Dekaliter,	=2.64
Prussia, ...1	Quart,	= .30
Austria, ...1	Maas,	= .37
Sweden, ...1	Kanna,	= .69
Denmark, ...1	Kande,	= .51
Switzerland, 1	Pot,	= .40
Turkey, ...1	Almud,	= 1.38
Mexico, ...1	Fasco.	= .63
Brazil, ...1	Medida,	= .74
Cuba,........1	Arroba,	= 4 01
South Spain, 1	Arroba,	= 4.25

COMPARISON OF GRAIN MEASURES.

Country.		U. S. Bushels.
England, ...1	Bushel,	= 1.031
France, ...1	Hectoliter	=2.84
Prussia, ...1	Scheffel,	= 1.56
Austria, ...1	Metze,	= 1.75
Russia 1	Chetverik	= .74
Greece, ...1	Kailon,	=2.837
Germany, ...1	Schef.	=1.5 to 3
Persia, ...1	Artaba,	= 1.85
Turkey, ...1	Kilo,	= 1.03
Brazil, ...1	Fan,	= 1.5
Mexico, ...1	Alque,	= 1.13
Madras, ...1	Parah,	=1.743

COMPARATIVE TABLE OF POUNDS IN DIFFERENT COUNTRIES

Austria, 100 lbs.	123.50	U. S.
Bavaria, "	123.50	"
Belgium, "	103.35	"
Bremen, "	110.12	"
Berlin, "	103.11	"
Denmark, "	110.00	"
Ger. Zoll. States,	110.25	"
Hamburg,	110.04	"
Nederland, 100 lbs.	108.93	U. S.
Portugal, "	101.19	"
Prussia, "	110.25	"
Russia, "	90.00	"
Spain, "	101.44	"
St. Domingo, "	107.93	"
Trieste, "	123.60	"

COMPARISON OF COMMERCIAL WEIGHTS.

Country.	Weight.	U. S. Lbs.
Austria,	1 Pfund,	= 1.23
Arabia,	1 Maund.	= .3
Brazil,	1 Arratel,	= 1.02
China,	1 Catty,	= 1.33
Denmark.	1 Pund,	= 1.10
East Indies,	1 Seer.	= 2.06
Egypt,	1 Rottoli,	= 1.008
France,	1 Kilogram,	= 2.20
Germany,	1 Pfund,	= 1.10
Mexico,	1 Libra,	= 1.02
Madras,	1 Vis,	= 3.125
Persia,	1 Rattel,	= 2.116
Russia,	1 Funt,	= .90
Sweden,	1 Pund,	= .93
Spain,	1 Libra,	= 1.016
Sicily,	1 "	= .7
Turkey,	1 Oka,	= 2.82
Japan,	1 Kin,	= .62

	Troy.	Apothecaries.	Avoirdupois.
1 Pound	= 5760 grains	= 5760 grains	= 7000 grains.
1 Ounce	= 480 "	= 480 "	= 437.5 "
	175 Pounds	= 175 pounds	= 144 pounds.

RAILROAD FREIGHT.—TABLE OF GROSS WEIGHTS.

When the actual weights are not known, the articles are billed as per the following table.

Ale and Beer,	320 lb.	per bbl.
" " "	170 "	" ½ "
" " "	100 "	" ¼ "
Apples, dried,	24 "	" bu
" green,	50 "	" "
" "	150 "	" bbl.
Beef,	320 "	" "
Bran,	20 "	" bu.
Brooms,	40 "	" doz.
Cider,	350 "	" bbl.
Charcoal,	22 "	" bu.
Eggs,	200 "	" bbl.
Fish,	300 "	" "
Flour,	200 "	" "
Highwines,	350 "	" "
Lime,	200 lb.	per bbl.
Malt,	38 "	" bu.
Millet,	45 "	" "
Nails,	108 "	" keg.
Oil,	400 "	" bbl.
Peaches, dried,	33 "	" bu.
Pork,	320 "	" bbl.
Potatoes (com.)	150 "	" "
Salt, Fine,	300 "	" "
" Coarse,	350 "	" "
" in Sack,	200 "	" "
Turnips,	56 "	" bu.
Vinegar,	350 "	" bbl.
Whiskey,	350 "	" "
One Ton Weight, U.S.	2000 lb.	

CU.FT.	CU. IN.		CU.FT.	CU. IN.	
.0167	28.875	4 Gills,..1 Pint.	11.229	19404	2 Tiecs..1 Punsh'n.
.0334	57.75	2 Pints..1 Quart.	4.2109	7276.5	31½ Gals.....1 Bbl.
.13368	231	4 Qts.,..1 Gallon.	8.421	14553	2 Bbls.......1 Hhd.
1.3368	2310	10 Gals...1 Anker.	16.84	29106	2 Hhds......1 Pipe.
2.406	4158	18 Gals...1 Runlet.	33.68	58212	2 Pipes......1 Tun.
5.614	9702	42 Gals...1 Tierce.			

The U. S. Standard Gallon contains 231 cubic in.=8⅓ lbs. avoirdup's.
" Imperial " " 277.274 " =1.2 U. S. gallons.
" old Beer Measure " " 282 "

In measuring tanks, reservoirs, etc., it will be sufficiently accurate to regard one cubic foot=7½ U. S. or 6¼ Imperial gallons.

Cubic feet × 100 ÷ 4 = quarts Imperial nearly.

The contents of a circular tank, in barrels of 31½ gallons,=the square of the diameter (in ft.) multiplied by the depth, mul. by .1865.

The number of U. S. gallons in a round tank=the square of the diameter, in feet,×by the depth×5⅞.

The number of U. S. gallons in a round tank, wider at one end than the other,=⅓ the sum of the squares, plus the product of the two end diameters, in feet,×the depth×5⅞.

For Imperial gallons×4.9 or 4.89469.

GAUGERS' WORK.

To find the contents of a cask in gallons.

RULE. Add two-thirds the difference of the head and bung diameters to the head diameter, then multiply the square of the sum by the length, all in inches, and the product for U.S. Gallons, by .0034, for Imperial Gallons, multiply by .0028325 for Ale Gallons, multiply by .0028.

NOTE. If the staves are but little curved, add .6 instead of ⅔.

How many U.S. gallons in a cask, length 40 in., head diameter 21 in. and bung diameter 30 in.?

$\therefore 21+(\overline{30-21}\times\frac{2}{3})=27$ in. mean diameter.

$\therefore 27^2\times40\times.0034=99.144$ gallons, Ans.

Or $27^2\times40\times\cdot0028\frac{1}{3}=82\cdot62$ Imperial gallons.

DRY MEASURE, IMP. AND U. S. STANDARD.

For measuring Grain, Fruit, Roots, Coal, etc.

IMP. CU. IN.	U. S. CU. IN.	PT.	QT.	GAL.	PK.	BU.	CM.	QR.		
34.659	33.60	...1	.....	......	....	....	....	....	..1	Pint.
69.318	67.20	2	1	......	....	....	....	....	..1	Quart.
277.274	268.80	8	4	1	....	....	....	....	..1	Gallon.
554.548	537.60	16	8	2	..1	....	....	....	..1	Peck.
2218.192	2150.42	64	32	8	4	..1	....	....	..1	Bushel.
8872.768	8601.68	256	128	32	16	4	..1	....	..1	Coomb.
17745.536	17203.36	512	256	64	32	8	2	..1	..1	Quarter.
70982.144	68813.44	2048	1024	256	128	32	8	4	..1	Chaldron.
	77415.12	2304	1152	288	144	36	OF COAL		..1	Chaldron.

The U. S. Standard Bushel contains 2150.42 cubic inches.
The Imperial English " " 2218.192 " "
A cylinder 18½ inches in diameter, 8 inches deep= 1 Bushel. U.S.
5 Stricken measures= 4 heap measures.

Cubic feet ×.8 = U. S. bushels nearly; add 44.5 for every 10,000 bushels.

Cubic feet ×.779 = Imperial bushels nearly.
Imperial Bushels×1.03152 the Product=U. S. Bushels.
U. S. Bushels×.969444 the Product=Imperial Bushels.

Any three factors that will produce the number of inches in a given quantity, will be the inside dimensions of a box to hold that quantity; hence a box 11.2×16×12 in., will contain 1 Standard Bushel. 924 cu. inches = 4 Liquid Gallons; therefore a box 12×7×11 inches will contain 4 gallons.

An open box made with the greatest economy of material; the altitude= the radius of the Base; if with a cover the altitude= the base.

The number of bushels + ¼ = the number of cubic feet.

The number of cubic feet—1-5=the number of Bushels. U. S.

The price per cental=the price per bushel×100÷the number of pounds in the bushel. See page 75

It is usual—with some local exceptions—to estimate the number of pounds to the bushel, as follows: Bran, 20 lbs.; Oats, 32 lbs; Barley, 48 lbs.; Maize and Rye, 56 lbs.; Wheat, Beans, Clover Seed and Potatoes, 60 lbs.

Diamond Weight.		Assayers' Weight.	
16 Parts	= 1 Grain.	240 Grains	= 1 Carat.
4 Grains	= 1 Carat.	2 Carats	= 1 Ounce.
1 Carat	= 31-5 Troy grs. (nearly.)	24 Carats	= 1 Pound.

The term Carat is also employed in estimating the fineness of Gold and Silver; when perfectly pure the metal is said to be "24 Carats fine." English Gold coin is 22 carats fine, that is, it consists of 22-24 pure gold, and 2-24 alloy.

To compute the fineness in thousandths, and the weight in ounces and thousandths is simpler, and admits of very minute subdivisions with great facility.

The coining of gold or silver does not change the REAL value of either; it stamps each piece of metal with a national, official certificate of its weight and fineness.

From one Troy pound of gold 22 carats, or .916 2-3 fine 46 29-40 Sovereigns are made, each weighing 123.27448 grains = 113.001605 grains of fine gold = $4.866563.

1 ounce of U. S. Standard Gold	=	$18.60465 =	£3.8230 =	£3,,16,, 5½
1 " " British " "	=	18.94918 =	3.8938 =	3,,17,,10½
1 " " Pure "	=	20.67184 =	4.248 =	4,, 4,,11½

Thousandths of an ounce ÷ 100 × 48 = grains.

Grains × 100 ÷ 48 = thousandths of an ounce.

U. S. Standard ounces of Gold ÷ .05375 = U. S. Dollars.

U. S. Gold Dollars × .05375 = Standard ounces.

To multiply by .05375, remove the point one place to the left and divide by 2, divide this quotient by 20, and the second quotient by 2; the sum of the quotients is the answer.

To find the value of any quantity, Troy weight in £ Sterling.

RULE. Regard each ounce as £1. the pennyweights as shillings, and half the grains as pence, and multiply by the pounds and parts of £1 per oz.

Find the cost of 3 oz. 7 pwts. 18 grs. of gold @ £3 17s. 6d. per oz.

£3 7s. 9d. × $3\frac{7}{8}$ = £13 2s. 6¼d. or 3·3875 × 3·875 = £13·1265.

Find the value of 11 oz. 13 pwts. 12 grs. of silver @ 4s. per. oz.

£11·675 × ·2 = 2·335 = £2 6s. 8½d. or £11 13s. 6d × $\frac{1}{5}$ = £2 6s. 8½d.

480 grains = 1 oz. troy. 480 halfpence = £1 sterling.

The number of halfpence in the price of a grain regarded as pounds sterling = the price of one ounce.

An ounce of gold at 3 halfpence a grain = £3 0s. 0d.

The number of pounds sterling in the price of an ounce regarded as halfpence = the price of one grain.

1 grain of gold at £4 per oz. = 4 halfpence = 2d.

The Gold Talent of Scripture = £5464 5s. 8d. = $26592·28; its ratio to the Silver Talent was 16 to 1 = the ratio of U.S. gold and silver coin.

The weight of gold, in ounces, and the fineness being given, to find its value in U. S. Gold Coin.

RULE. Multiply the weight by twice the fineness, multiply by 10 and divide the product by 30, and the quotient by 129; the sum of the product and the quotients is the answer.

Or multiply the given weight by the fineness × 1000 × 8, and divide the product by 387.

$$\overline{1\times.9\times1000\times8}\div387 = 18.60465.$$

The fineness and weight of Silver being given, to find its value in U. S. Silver dollars 9-10 fine, 412½ grains weight.

RULE. For pure silver, if in grains, divide by 9×10×11×3 and multiply by 8, or divide by .9×412.5.

EXAMPLE. Pure silver, grains 371.25×8÷9×10×11×3= $1.

If in ounces, divide the weight and fineness by .9 × .895375.

Or multiply the given weight by the fineness and by 1.28; repeat the figures in the product, under, and two places to the right, as often, and to as many decimal places as the answer requires; the sum is the answer.

EXAMPLE. Find the value in silver dollars of 1 oz. of silver 9-10 fine.

$$\begin{array}{r} 1\times.9\times1.28 = 1.152 \\ 1152 \\ 1152 \\ \hline \$1.1636352 \end{array} \text{ Ans.}$$

To make a compound of any weight and fineness.

RULE. Divide the fineness sought by the fineness to be alloyed; the quotient is the weight required to make a compound of one ounce of the desired fineness.

EXAMPLE. Required to make a compound of one ounce 14 carats fine by alloying gold 22 carats fine.

14 ÷ 22 = .63636 gold + .36364 alloy = 1 ounce.

To find how many ounces of a lower fineness must be added to one ounce of a higher fineness to make a compound of any given fineness.

RULE. Divide the difference of the two higher by the difference of the two lower finenesses.

EXAMPLE. Required a compound of 14 Carats fine by mixing 12 carat fine with 21 carat fine.

$$\frac{21-14=7}{14-12=2} = 3\tfrac{1}{2}.$$ 3½ oz. 12 fine + 1 oz. 21 fine = 4½ oz. 14 Carat fine.

At the English Mint one troy pound of standard silver 9¼ fine is coined into 66 shillings. 1 oz. of standard silver contains 444 grains of pure silver. The U.S. silver dollar is ·9 fine and weighs 412½ grains= 371¼ grains of pure silver.

English standard silver being worth 54d. an oz., and the £ being worth $4·86 U.S. gold, what is the intrinsic value of the U.S. silver dollar in U.S. gold coin? Ans. $0·915.

54d.=£0·225. ·225×4·86=1·0935. $\frac{1{\cdot}0935\times371\frac{1}{4}}{444}={\cdot}915.$

The intrinsic value of the U.S. silver dollar in U.S. gold coin=the market price in pence×·017; its value in pence=the market price in pence ×·836.

Silver being 54d. an oz. the U.S. silver dollar=54×·017=$·918.

Silver being 54d. an oz. the U.S. silver dollar=54×·836=45·14d.

SEC.	MIN.	HRS.	DA.	WK.		
60	1	......	......	....	..1	Minute.
3600	60	1	......	....	..1	Hour,
86400	1440	24	1	....	..1	Day,
604800	10080	168	7	..1	..1	Week,
31536000	525600	8760	365	52	..1	Common Year,
31622400	527040	8784	366	....	..1	Leap Year.

TIME is a measured portion of duration, the unit of which is the mean solar day.

12 Calender months = 13 lunar months = 1 year.

365 days, 5 hrs. 48 minutes, 50 seconds = 1 Solar year.

10 years = 1 decade. 10 decades = 1 century.

400 years = 146,097 days, a number exactly divisible by 7.

The civil day begins and ends at 12 o'clock, Midnight.

The Astronomical day begins and ends at 12 o'clock, Noon.

As the year contains 365¼ days, nearly, we reckon three years in every four as containing 365 days, and the fourth, leap year, as containing 366 days; the leap year is always a multiple of 4.

The even centuries not divisable by 400 are not leap years.

Formerly the new year began on the 25th of March and was so reckoned in England until 1753.

In ordinary business computations, 1 year = 12 mos. = 360 ds. 1 month = 30 days.

+1	−2	+1		+1		+1	+1		+1		+1
Jan.	Feby.	Mar.	Apl.	May,	June,	July,	Aug.	Sept.	Oct.	Nov.	Dec.

In the common year February has two days less than 30, in leap year 1 day less; seven months have one day more.

To find the exact number of days between two dates.

Multiply the number of entire months by 3, call the product tens; add the extra days, and 1 day for each month of 31 days; when Feb'y occurs, deduct 2 days for the common, and 1 day for Leap year.

How many days from 1st of the 4th month to 9th of the 11th month.
11 mo.—4 mo. = 7 mo. $\overline{7\times30}+9+4=223$ days.

Proof spirit is a spirit that at the temperature of 51° of Fahrenheit's thermometer weighs exactly $\frac{12}{13}$ parts of an equal measure of distilled water: by volume, or measure, it contains 57·06 per cent. of absolute alcohol, and its specific gravity at 60 Fahrenheit is ·920.

The terms *Proof*, *Over Proof*, (O.P.), and *Under Proof* (U.P.) are used to indicate alcoholic values; in calculations relating to these values the unit 1=*Proof;* the O.P. strengths are added to and written as decimals of that unit; thus 25 O.P.=1·25, and one gallon of such spirit=1·25 or $1\frac{1}{4}$ gallons of *Proof* spirit.

The U.P. strengths are deducted from and written as decimals of the unit, thus 20 u.p. $=\overline{1 - \cdot 20} = \cdot 80$, and each gallon of such spirit will contain 8 parts *Proof* spirit and 2 parts water.

To find how much of each of two given qualities are required to make a mixture of a given intermediate quality.

The difference of the intermediate and the higher strengths= the quantity required of the lower strength, and the difference of the lower and the intermediate strengths=the quantity required of the higher strength.

Having spirits 20 O.P. and 20 u.p., find how many gallons of each are required to make a mixture= *Proof.*

$$\begin{array}{llll} 20\text{ O.P.}=120 & 1{\cdot}20 - 1 = {\cdot}20 & \frac{\cdot 20}{\cdot 20} = \frac{1}{1} = 1 \text{ gallon of each.} \\ 20\text{ U.P.}\quad 80 & 1{\cdot}00 - {\cdot}80 = {\cdot}20 & \end{array}$$

1 gall. strength 1·20 + 1 gall. ·80 = 2·00 = 2 galls. of Proof.

Having spirit =20 O.P. How much spirit and how much water are required to make a mixture=20 U.P.?

$$\begin{array}{ll} 1{\cdot}20 - {\cdot}80 = {\cdot}40 & \frac{\cdot 40}{\cdot 80} = \frac{1 \text{ gall. of water.}}{2 \text{ galls. of 20 O.P.}} \\ {\cdot}80 - \ \ 0 = {\cdot}80 & \end{array}$$

20 O.P.=1·20 2 galls. of 1·20=2·40 galls. of Proof.
20 U.P.= ·80 3 „ „ ·80=2·40 „ „ „

How many gallons of spirit at 8 u.p. are required to reduce 30 gallons of spirit 60 O.P. to 5 O.P.?

1·60 − 1·05 = ·55
1·05 − ·92 = ·13 30 × 55 ÷ 13 = 127 galls. Ans.

127 × 92 = 11684
30 × 160 = 4800

157 16484 157 × 1·05 = 164·85.

or 30 galls. 60 O.P.= 48 galls. of proof spirit.
127 „ 8 U.P.=116·8 „ „ „ „

157 „ 5 O.P.=164·8 „ „ „ „

MISCELLANEOUS.

How many strokes does a clock strike in 12 hours?

$$\frac{\overline{12+1}\times 12}{2}=78 \text{ strokes.}$$

How many barrels in a triangular pile, 49 barrels at the base and 1 at the top?

$$\frac{\overline{49+1}\times 49}{2}=1225 \text{ barrels.}$$

O'Leary with ten tramps have two days start, and make 8 miles a day; how long will it take Rowell with 5 trampers travelling 10 miles a day to overtake O'Leary and his men?

Miles $10-8=2.\quad \frac{8\times 2}{2}=8$ days. Ans.

The sum of two numbers is 140; the larger is to the smaller as 1 to $\frac{5}{9}$, what are the numbers?

$$\frac{9}{9}+\frac{5}{9}=\frac{14}{9}\qquad \left.\begin{array}{l}140\times\frac{9}{14}=90\\ 140\times\frac{5}{14}=50\end{array}\right\}=140$$

A Bin 9 ft. 6 in. long, 6 ft. wide, 4 ft. 3 in. deep, will hold how many Imperial bushels.

$\frac{19}{2}\times\frac{6}{1}\times\frac{17}{4}\times\frac{8}{10}-4.845=188.955$ bushels. Ans.

NOTE. The imperial bushel is 2218.192 Inches, ten eighths of a foot, nearly, deduct $2\frac{1}{2}$ from every 100 bushels in the product, this result multiplied by 8 will be the number of Imp. gallons,

What is the cost of 732 lbs. of Coal at \$14. per ton, 2240 lbs. to the ton?

$$\frac{732\times 14}{8\times 4\times 7}=\$4.575. \text{ Ans}$$

A bin 9 ft, 6 in. long, 6 ft. wide, and 4 ft. 3 in. deep is full of wheat, what is its value at $2.05 a bushel?

$\frac{19}{2}\times\frac{6}{1}\times\frac{17}{4}\times\frac{8}{10}+.87\times2.05=\$399.07.$ Ans.

Note. The standard bushel is 2150.42 inches; ten-eighths of a foot, nearly, the difference is .44 bu. in each 100 *R.*259.

Divide £1 into 3 parts in the proportion of A, $\frac{1}{2}$, B, $\frac{1}{3}$, C, $\frac{1}{4}$. $\frac{12}{2}+\frac{12}{3}+\frac{12}{4}=6+4+3=13.$

Ans. $\frac{6}{13}, \frac{4}{13}, \frac{3}{13}.$

How many cubic feet in a case 3 ft. 6 in. by 2 ft 8 in. by 1 ft. 10 in?

$\frac{7}{2}\times\frac{8}{3}\times\frac{11}{6}=17\frac{1}{9}$ ft. Ans.

If 7 cats, kill 7 rats, in 7 minutes, how many cats will kill 100 rats in 50 minutes?

$\frac{7\times7\times100}{7\times50}=14.$ Ans. 14 cats.

If it cost $24 to carry 6 tons 20 miles, what will it cost to carry 12 tons 120 miles?

$\frac{24\times12\times120}{6\times20}=288.$ Ans. $288.

How many bricks will pave a walk 200 ft. long, by 16 feet; bricks 8 in. by 4 in.; also if the bricks measure $9\times4\frac{1}{2}$ in.

$\frac{200\times16\times3\times3}{2\times1}=14,400$ $\frac{200\times16\times4\times8}{3\times3}=11,377$

How many bricks $9\times4\frac{1}{2}\times3$ inches are required to build a wall $160\times20\times1\frac{1}{2}$ feet ?

$\frac{160\times20\times3\times4\times8\times4}{2\times3\times3\times1}=68,266$ bricks.

How many shingles for a roof 60 ft. long, rafters 20 feet, two sides, shingles to show 6×4 inches.

$\frac{60\times20\times2\times2\times3}{1\times1}=14,400$ shingles.

Multiply 66 by $\frac{2}{3}$: $22\ \dfrac{\cancel{66} \times 2}{\cancel{3}} = 44.$

Divide 66 by $\frac{2}{3}$: $33\ \dfrac{\cancel{66} \times 3}{\cancel{2}} = 99.$

Divide $168 \times 2 \times 7$ by 7×3: $\dfrac{\cancel{7} \times 2 \times \overset{56}{\cancel{168}}}{\cancel{7} \times \cancel{3}} = 112.$

Divide £99 amongst 3 persons, A to have $\frac{5}{11}$, B $\frac{4}{11}$, and C $\frac{2}{11}$.

$\cancel{11} \left| \begin{array}{l} \cancel{99}\,9 \\ 5 \end{array} \right.$ $\cancel{11} \left| \begin{array}{l} \cancel{99}\,9 \\ 4 \end{array} \right.$ $\cancel{11} \left| \begin{array}{l} \cancel{99}\,9 \\ 2 \end{array} \right.$ A £45, B £36, C £18.

Two merchants load a ship with goods worth £5000, A owns £3500, and B the rest; the goods suffer damage valued at £1000, what is each man's share of the loss?

$5\cancel{000} \left| \begin{array}{l} 1\cancel{000} \\ 3500 \end{array} \right.$ $5\cancel{000} \left| \begin{array}{l} 1\cancel{000} \\ 1500 \end{array} \right.$ A loses £700.
B ,, £300.

B and C gain by trade £182; B put in £300, and C £400, what is the gain of each?

$7\cancel{00} \left| \begin{array}{l} 3\cancel{00} \\ 182 \end{array} \right.$ $7\cancel{00} \left| \begin{array}{l} 4\cancel{00} \\ 182 \end{array} \right.$ B £78.
C £104.

A person owning $\frac{3}{5}$ of a mine sells $\frac{3}{4}$ of his share for £1710, what is the value of the whole mine?

$$190\ \frac{\cancel{1710} \times 4 \times 5}{\cancel{3} \times \cancel{3}} = £3800.$$

How much money will buy $\frac{3}{4}$ of $\frac{3}{5}$ of a mine worth £3800?

$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$ $\dfrac{3800 \times 9}{20} = £1710.$

If $\frac{1}{3}$ of 6 be 3, what will $\frac{1}{4}$ of 20 be?

$$\frac{3 \times \cancel{3} \times \cancel{20}\,5}{2\ \cancel{6} \times \cancel{4}} = 7\tfrac{1}{2}.$$

A compositor can set 20 pages in $\frac{2}{5}$ of a day, another could set 20 pages in $\frac{3}{4}$ of a day, how long will it take the two men working together to do the work?

$$\frac{4}{3}+\frac{5}{2}=\frac{23}{6} \qquad \frac{23}{6}\text{ inverted } = \frac{6}{23}\text{ of a day.}$$

A cistern has 5 faucets; the first will fill it in 1 hour, the second in two, the third in 3, the fourth in 4, and the fifth in 5 hours; in what time will the cistern be filled, all the faucets running at once?

$$\frac{60}{1}+\frac{60}{2}+\frac{60}{3}+\frac{60}{4}+\frac{60}{5}=137. \text{ Ans. } \frac{60}{137} \text{ of an hour.}$$

A says to B, give me \$7 and I shall have as much money as you; B replies, give me \$7 and I shall have twice as much as you; how much money had each?

$7\times5=35 \quad 7\times7=49 \quad$ A \$35, B \$49.

How many different pairs can be made with 7 units?

$$\frac{7\times6}{2}=21 \text{ pairs.}$$

A cubic foot of iron weighs 7680 oz., or 480 lbs.
A sheet of iron 1ft. square and 1in. thick, weighs 40lbs.
Find the weight of a sheet of iron $6\frac{1}{4}$ feet long, 3 ft. 4 in. wide, and $\frac{1}{8}$ in. thick.

$$\frac{25\times10\times1\times480}{4\times3\times96} \text{ or } \frac{25\times10\times5}{4\times3}=104\tfrac{1}{6} \text{ lbs.}$$

The usual discount off the selling price being 40 per cent. what increase must be made in the selling price to obtain the same net return and allow 55 °/ₒ discount?

$$\begin{array}{l}100-40=60\\100-55=45\end{array}=1{\cdot}33\tfrac{1}{3}. \qquad \text{Ans. } \tfrac{1}{3}.$$

If $21\frac{3}{4}$ bushels of oats will seed $9\frac{2}{3}$ acres, how many bushels will seed 100 acres?

$$\frac{87\times3\times100}{4\times29}=225 \text{ bushels.}$$

How many 12ths are there in ·75?

$$\cdot75\times12=9. \quad \cdot75=\tfrac{9}{12}.$$

A piece of cloth 50 yds. long and 43 inches wide, weighs 114 lbs.; find the weight of one square yd.

$$\frac{6\times6\times114}{50\times43}=1\cdot909 \text{ lbs.}=1 \text{ lb. } 14\tfrac{1}{2} \text{ oz.}$$

Proof, 50 yds.$\times$43 in. $=59\frac{26}{36}$ yds. $59\frac{26}{36}\times1\cdot909=114$.

$150 is due Jan. 1st., $78 is paid down, on July 1st., the account is settled by paying $78. What rate per cent is paid for the accomodation?

$$\$150-78=\$72. \ \frac{6\times2\times100}{72}=16\tfrac{2}{3} \text{ per cent.}$$

Find the value of an ounce of silver, gold being worth £3,,18,,7 per ounce, ratio $15\frac{1}{2}$ to 1. also 16 to 1.

£3,,18,,7 $\div15\frac{1}{2}=60\frac{26}{31}$d. £3,,18,,7$\div16=58\frac{15}{16}$d.

To find the amount in 365 days of any given number of pence per day: Multiply the given number of pence by $1\frac{1}{2}$, call it pounds, and add the product of the pence multiplied by 5.

Dice. The number of different combinations that can be made with any given number of dice is equal to a power of 6, equal to the given number of Dice.

How much is due to a man working 22 days, at $39 per month of thirty days; also 26 working days to the month?

$$\frac{39\times22}{30}=\$28\cdot60. \qquad \frac{39\times22}{13\times2}=\$33.$$

Find the charges on 1000 cases, each $16\times12\times6$ inches, at 16 shillings per ton of 40 feet.

$$\frac{1000\times4\times1\times1\times16}{3\times1\times2\times40}=266.67\text{s.}=£13\ 6\text{s}\ 8\text{d.}$$

TO WRITE AND READ BRITISH MONEY DECIMALLY.

To reckon Interest, Discount, Per Centages, &c., with facility, it is absolutely essential to be able, instantly, to write the fractions of British money in decimals of £1 stg.; also to read off those decimals in shillings, pence and farthings. The following exercises afford excellent practice: 2s. = £·1; 1s. = £·05; 6d. = £·025; 3d. = £·0125; 1½d. = £·00625; ¾d. = £·003125; ¼d. = £·001041$\dot{6}$. For the pence, write the farthings therein as thousandths of £1, and add 1 for each sixpence. See page 59.

¼d = ·001 + ·25d ÷ 6 = ·001041$\dot{6}$	6¼d = ·026 + ·25d ÷ 6 = ·026041$\dot{6}$
½d = ·002 + ·50d ÷ 6 = ·00208$\dot{3}$	6½d = ·027 + ·50d ÷ 6 = ·02708$\dot{3}$
¾d = ·003 + ·75d ÷ 6 = ·003125	6¾d = ·028 + ·75d ÷ 6 = ·028125
1d = ·004 + 1·00d ÷ 6 = ·0041$\dot{6}$	7d = ·029 + 1·00d ÷ 6 = ·0291$\dot{6}$
1¼d = ·005 + 1·25d ÷ 6 = ·005208$\dot{3}$	7¼d = ·030 + 1·25d ÷ 6 = ·030208$\dot{3}$
1½d = ·006 + 1·50d ÷ 6 = ·00625	7½d = ·031 + 1·50d ÷ 6 = ·03125
1¾d = ·007 + 1·75d ÷ 6 = ·007291$\dot{6}$	7¾d = ·032 + 1·75d ÷ 6 = ·032291$\dot{6}$
2d = ·008 + 2·00d ÷ 6 = ·008$\dot{3}$	8d = ·033 + 2·00d ÷ 6 = ·033$\dot{3}$
2¼d = ·009 + 2·25d ÷ 6 = ·009375	8¼d = ·034 + 2·25d ÷ 6 = ·034375
2½d = ·010 + 2·50d ÷ 6 = ·01041$\dot{6}$	8½d = ·035 + 2·50d ÷ 6 = ·03541$\dot{6}$
2¾d = ·011 + 2·75d ÷ 6 = ·011458$\dot{3}$	8¾d = ·036 + 2·75d ÷ 6 = ·036458$\dot{3}$
3d = ·012 + 3·00d ÷ 6 = ·0125	9d = ·037 + 3·00d ÷ 6 = ·0375
3¼d = ·013 + 3·25d ÷ 6 = ·013541$\dot{6}$	9¼d = ·038 + 3·25d ÷ 6 = ·038541$\dot{6}$
3½d = ·014 + 3·50d ÷ 6 = ·01458$\dot{3}$	9½d = ·039 + 3·50d ÷ 6 = ·03958$\dot{3}$
3¾d = ·015 + 3·75d ÷ 6 = ·015625	9¾d = ·040 + 3·75d ÷ 6 = ·040625
4d = ·016 + 4·00d ÷ 6 = ·016$\dot{6}$	10d = ·041 + 4·00d ÷ 6 = ·041$\dot{6}$
4¼d = ·017 + 4·25d ÷ 6 = ·017708$\dot{3}$	10¼d = ·042 + 4·25d ÷ 6 = ·042708$\dot{3}$
4½d = ·018 + 4·50d ÷ 6 = ·01875	10½d = ·043 + 4·50d ÷ 6 = ·04375
4¾d = ·019 + 4·75d ÷ 6 = ·019791$\dot{6}$	10¾d = ·044 + 4·75d ÷ 6 = ·044791$\dot{6}$
5d = ·020 + 5·00d ÷ 6 = ·0208$\dot{3}$	11d = ·045 + 5·00d ÷ 6 = ·0458$\dot{3}$
5¼d = ·021 + 5·25d ÷ 6 = ·021875	11¼d = ·046 + 5·25d ÷ 6 = ·046875
5½d = ·022 + 5·50d ÷ 6 = ·02291$\dot{6}$	11½d = ·047 + 5·50d ÷ 6 = ·04791$\dot{6}$
5¾d = ·023 + 5·75d ÷ 6 = ·023958$\dot{3}$	11¾d = ·048 + 5·75d ÷ 6 = ·048958$\dot{3}$
6d = ·025 = ·025	12d = 050

To show the number of farthings in any sum of British money written as decimals of £1, remove the point 3 places to the right, then multiply by 4, place the product 2 places to the right, and subtract. The difference shows the number of farthings.

Examples: 3¾d. = ·015·625 × ·04.　　7¼d. = ·030·208$\dot{3}$ × ·04

62500　　1 208333

Farthings 15·000　　Farthings 29·0000

11¾d. = ·048·958$\dot{3}$ × ·04　　£37 7s. 4¼d. = 37·367·708$\dot{3}$ × ·04

1 958333　　1 494 708333

Farthings 47·0000　　Farthings 35 873·0000

For Reckoning Simple Interest; also see page 47.

The sum of money upon which Interest is reckoned is called the Base, or the Principal. 1 per cent. means 1 to be taken for each 100 in the Principal; 2 per cent. 2 for each 100; 5 per cent. 5 for each 100. &c., &c.

TO RECKON INTEREST AT 5 % PER ANNUM.

The mutual adaptiveness of the fractions of *Time* and the fractional *Currency* greatly facilitates the reckoning of interest on British money at the legal rate of interest, 5 per cent. 5 % is equal to $\frac{1}{20}$ of any sum. 1s. = $\frac{1}{20}$ of £1, therefore any number of *Pounds* regarded as *Shillings* = the interest for 1 year at 5 % per annum. The interest for 1 year at 5 % on £100 = 100s. = £5, twelve calendar months = 1 year, 12 *Pence* = 1 *Shilling*, therefore any number of *Pounds* regarded as *Pence* = the interest for one month at 5 per cent. The interest on £100 for 1 month = 100d. = 8s. 4d. Pointing off the right hand figure of any number divides that number by 10. $\frac{1}{10}$ or .1 of 30 days = 3 days, therefore to find the interest on any number of pounds sterling for 3 days at 5 % point off the right hand figure in the pounds and the figures then shew the interest, in *Pence*, for 3 days at 5 %, the interest on £100 for 3 days at 5 % = 10·0d. = 10d.

When interest is found in this way for any given number of days, if read off literally, there will be an excess in the answer = 1d. for each 6s. The excess arises in this way: 12 months of 30 days = 360 days—there are 365 days in the common year, $\frac{1}{360}$ of anything is a little more than $\frac{1}{365}$, then to compensate for the difference of the 365ths and the 360ths, deduct 1 *Penny* for each 6 *Shillings* in the answer. On interest for the even *Months* no deduction need be made.

The interest on £248 4s. 9d. for 1 month at 5 % = 248¼d. = £1 0s. 8¼d.

When the shillings and pence in the principal are nearly equal to 5s. their interest for one month = ¼d., when near 10s. = ½d., when near 15s. = ¾d., when more than 15s. regard the interest as = 1d.

The interest at 5 % for—

1 month on £55 5s. 3d. = 55¼d. = 4s. 7¼d.
3 days on £55 5s. 3d. = 5·5d. = 5½d.
1 month on £255 5s. 3d. = 240d. + 15¼d. = £1 1s. 3¼d.
3 days on £255 5s. 3d. = 25·5 − ¼d. = 2s. 1¼d.
1 month on £495 5s. 3d. = 480d. + 15¼d. = £2 1s. 3¼d.
3 days on £495 5s. 3d. = 49·5d. − ¾d. = 4s. 0¾d.
1 month on £745 15s. 2d. = 720d. + 25¾d. = £3 2s. 1¾d.
3 days on £745 15s. 2d. = 74·5d. − 1d. = 6s. 1½d.

TO RECKON INTEREST ON POUNDS STERLING AT 6 % PER ANNUM.

The interest on £100 for 1 year at 6 % = £6 = 120 shillings; 12 months = 1 year, $\frac{1}{12}$ the interest for 1 year = the interest for 1 month, $\frac{1}{12}$ of 120s. = 10s., 10s. = $\frac{1}{10}$, or ·1 of 100s., therefore to find the interest for 1 month at 6 % on any number of pounds sterling point off the right hand figure in the pounds and then regard the figures as shillings.

The interest for 1 month at 6 % on £100 = 10·0 = 10s., 3 days = $\frac{1}{10}$, or ·1 of 30 days, therefore to find the interest for 3 days at 6 % on any number of pounds sterling point off the two right hand figures in the pounds and regard the figures as shillings.

The interest at 6 % for—

3 days on £100 = 1·00 = 1s.
1 month on £133 6s. 8d. = $13{\cdot}\dot{3}\dot{3}$s. = 13s. 4d.
3 days on £133 6s. 8d. = $1{\cdot}3\dot{3}$s. = 1s. 4d.

To read off Decimals of a Shilling in Pence.

Multiply by 12, commencing with the 2nd figure to the right of the point, add the carrying figure to the product of the first figure to the right of the point and divide by 10.

$\cdot 25\text{s.} = \overline{2 \times 12} + 6 = 30 = 3{\cdot}0\text{d.} = 3\text{d.}$

$\cdot 29\text{s.} = \overline{2 \times 12} + 11 = 35 = 3{\cdot}5\text{d} = 3\tfrac{1}{2}\text{d. nearly.}$

$\cdot 89\text{s.} = \overline{8 \times 12} + 11 = 107 = 10{\cdot}7\text{d.} = 10\tfrac{3}{4}\text{d. nearly.}$

$\cdot 75\text{s.} = \overline{7 \times 12} + 6 = 90 = 9{\cdot}0\text{d.} = 9\text{d.}$

To read off Decimals of a Penny in Farthings.

Proceed as above, using 4 as a multiplier.

$\cdot 75\text{d.} = \overline{7 \times 4} + 2 = 3{\cdot}0 = \tfrac{3}{4}\text{d.}$ $\cdot 25\text{d.} = \overline{2 \times 4} + 2 = 1{\cdot}0 = \tfrac{1}{4}\text{d.}$

To read off 10 *per cent. of Shillings and Pence.*

Multiply the unit figure in the shillings by 12. Add the pence, and remove the point one place left; regard the 1, if any, in the tens place as 1s.

$10\ \%\text{ of 7s. 6d.} = \overline{\cdot 7 \times 12} + 6\text{d.} = 9{\cdot}0\text{d.} = 9\text{d.}$

$10\ \%\text{ of 8s. 9d.} = \overline{\cdot 8 \times 12 + 9\text{d.}} = 10{\cdot}5\text{d.} = 10\tfrac{1}{2}\text{d.}$

$10\ \%\text{ of 17s. 11d.} = 1\text{s.} + \overline{\cdot 7\text{d.} \times 12} + 1{\cdot}1\text{d.} = 9{\cdot}5 = 9\tfrac{1}{2}\text{d.}$

To read off 5 *per cent. of Shillings and Pence.*

Multiply the shillings by 6. Add half the pence and point off 1 place to the left.

$5\ \%\text{ of 7s. 6d.} = \overline{7 \times 6} + 3 = 4{\cdot}5 = 4\tfrac{1}{2}\text{d.}$

$5\ \%\text{ of 8s. 9d.} = \overline{8 \times 6} + 4\tfrac{1}{2} = 5{\cdot}25 = 5\tfrac{1}{4}\text{d.}$

To Construct Formulas for Reckoning Interest.

Bankers, Money-lenders, and others having often to reckon Interest on various sums for *one* given *Time* and *Rate* may easily construct a formula, or common multiplier, for *all sums* for that *Time* and *Rate.* This may be done by finding the interest on the *units* of time and value at the given rate. This method will be found better than the use of Tables, as it requires less time, and there is less risk of error.

The interest on £1 for 1 day at 1 % =

$$\frac{1 \times 1 \times 1}{100 \times 365} = £\cdot00002739726$$

The interest on any number of Pounds sterling for 1 day at 1 % = the given number of Pounds × ·00002739726.

The interest on £1000 for 1 day at 1 % = ·00002739726 × 1000 = ·000·02739726 = £·02739726 = 6½d. nearly.

To find a multiplier for any other time and rate: Multiply ·00002739726 by the number of days and the required rate.

Example—Wanted a common multiplier for all sums for 95 days at 5 % per annum.

$$\cdot00002739726 \times 95 \times 5 = \cdot013013698.$$

To find the interest on any sum of money for 95 days at 5 %. Multiply the principal by ·013013698, or by as much of the multiplier as is necessary to find the answer to three decimal places; the unit in the fourth decimal place is of less value than $\frac{1}{10}$ of a farthing.

A. The interest on £75 15s. 6d. for 95 days at 5 % = £75·775 × ·013013 = ·985 = £0 19s. 8½d.

$\frac{1}{10}$ or ·1 of the interest on any sum for any given time at 5 % = the interest on that sum at the ½ of 1 %, therefore it follows that $\frac{1}{10}$ or ·1 of any sum for any time at 5 % × twice any other rate = the interest on that sum for the latter rate.

The interest on £1 for 95 days at 5 % = £·013013698, then the interest on £1 for 95 days at 3½ % = ·0013013698 × 7 = £·0091095886.

B. The interest on £75 15s. 6d. for 95 days at 3½ % = £75·775 × ·0091095886 = ·691 = £0 13s. 10d.

Wanted a common multiplier for all sums for 93 days at 6 % per annum.

$$\cdot00002739726 \times 93 \times 6 = \cdot015287671.$$

C. The interest on £69 14s. 6d. for 93 days at 6 % = £69·725 × ·01528 = 1·066 = £1 1s. 3¾d.

To avoid encumbering the reckoning with needless figures, remove the decimal point in the principal as many places left, and one more, as there are ciphers to the left of the significant figures in the multiplier; use only so much of the multiplier—reversed as is necessary to find the answer to three decimal places. The unit in the fourth decimal place = less than $\frac{1}{10}$ of a farthing.

A.		*B.*		*C.*
·75·775		·075·775		·69·725
10 31		901 9		72 51
·75 77		·681 9		·69 72
·22 73		7 6		·34 86
7		6		1 39
·98 57		·690 1		49
				1·06 46

For this method of multiplying, see p. 36.

To read off decimals of a Pound in Shillings, Pence, and Farthings, regard each unit in the first decimal place as 2s., and a 5 in the second decimal place as 1s.; divide the remainder by 4, and write the quotient as pence, deducting ¼d. for each sixpence in the quotient.

£7·775 = £7 + 15s. + 6d. = £7 15s. 6d.

£48·8864583 = £48 + 17s. + 8¾d. = £48 17s. 8¾d.

In ordinary business computations 1 year = 12 months = 360 days. 1 month = 30 days. In the British Islands Interest is reckoned for *Days* on the basis of the exact number of days in the year; interest for each day is reckoned for $\frac{1}{365}$ of a year.

TO FIND THE DIFFERENCE OF TIME BETWEEN TWO DATES.

Rule. Subtract the earlier from the later date.

EXAMPLE.—For what time must Interest be charged on a debt due April 12th, 1882, and settled June 24th, 1883?

	83	6	24
Process,	82	4	12
	1	2	12

Ans. 1 yr. 2 mos. $\overline{12 + 1}$ days.

In the common year February has two days less than 30, in leap year one day less; seven months have one day more.

30+1	30−2	30+1	30	30+1	30	30+1	30+1	30	30+1	30	30+1
Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.

To find the exact number of days between two dates.

Learn to regard each month by its number instead of by its name:—Jan. 1st month, Feb. 2nd, Sept. 9th, &c., &c.

Multiply the number of entire months by 3, call the product tens, add the extra days, and 1 day for each month of 31 days; when February occurs deduct 2 days for the common and 1 day for Leap year.

How many days from 1st of the 4th month to 9th of the 11th month? 11 mo.—4 mo. = 7 mo. $\overline{7 \times 30} + 8 + 4 = 222$ days.

The Rules for these examples will be found on pp. 81, 82, *&c.*:-

1. The base of a triangle is 5 ft., and the altitude 2·4 ft. What is the area? $5 \times 1{\cdot}2 = 6$ ft.

2. The three sides of a triangle are 3, 4, and 5 ft. What is the area? 6 square ft.

$3+4+5 \div 2 = 6, \quad 6-3=3, \quad 6-4=2, \quad 6-5=1.$
$6 \times 3 \times 2 \times 1 = 36, \quad \sqrt{36} = 6$ sq. ft. Ans.

The three sides of a triangle are 9, 12, and 15 ft. What is the area? 54 square ft.

$9+12+15 \div 2 = 18, \quad 18-9=9, \quad 18-12=6, \quad 18-15=3.$
$18 \times 9 \times 6 \times 3 = 2916, \quad \sqrt{2916} = 54$ sq. ft. Ans.

3. The base of a triangle is 32 ft., the perpendicular 24 ft. Find the length of the hypothenuse.

$$\left.\begin{array}{l} 32^2 = 1024 \\ 24^2 = 576 \end{array}\right\} = 1600. \quad \sqrt{1600} = 40 \text{ ft. Ans.}$$

4. Prove by the next rule.

$32^2 = 1024, \quad 24+40=64, \quad 1024 \div 64 = 16$ ft. Ans.
$64+16 \div 2 = 40.$

5. A room is 4 yds. high, 8 yds. broad, and 12 yds. long. What is the length of a diagonal line from the upper to the opposite lower corner of the room? 15 yds.

$4^2 = 16, \quad 8^2 = 64, \quad 12^2 = 144, \quad 16+64+144 = 224.$
$\sqrt{224} = 15$, nearly.

6. The base of a triangle is 36 ft., and the hypothenuse 45 ft. Find the length of the other side.

$45^2 = 2025.$
$36^2 = 1296. \quad 2025 - 1296 = 729, \quad \sqrt{729} = 27$ ft. Ans.

7. The area of a triangle is 54 ft., and the base 12 ft. Find the altitude. The altitude of a figure is the straight line drawn from its vertex perpendicular to the base.

$54 \div 12 = 4{\cdot}5. \qquad 4{\cdot}5 \times 2 = 9$ ft. Ans.

8. The frustrum of a pyramid 12 ft. × 12 ft. at the base, 4 ft. × 4 ft. at the top, and 20 ft. high. Find how many board and how many cubic ft.

$12^2 = 144. \qquad 208 \times 20 \times 4 = 16{,}640$ bd. ft. Ans.
$4^2 = 16.$
$12 \times 4 = 48. \qquad \frac{1}{3}$ of $208 \times 20 = 1386\frac{2}{3}$ cu. ft. Ans.
$\overline{208}$

9. A wedge is 9×3 in. at the butt, 6 in. feather edge, and 2 ft. long. How many board ft.?

$\frac{3}{4} + \frac{3}{4} + \frac{1}{2} = 2$ ft.
$\frac{3}{4} + \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} \times 2 \times 12 \div 6 = 2$ ft. $\qquad 2 \times \frac{1}{4} \times 2 \times 2 = 2$ ft. Ans.

10. A wedge is 8 in. × 8 in. at the butt, 5 in. feather edge, and 4 ft. long. Find the solidity in board feet.

$$\overline{8+8+5} \times \overline{8\times4} \div 6\times12=9\tfrac{1}{3}\text{ bd. ft.}$$

How many board ft. in a wedge 25 ft. long, 8 in. × 8 in. butt, and 5 in. feather edge?

$$8+8+5=21. \qquad 21\times8=168.$$
$$168\times25\times\tfrac{1}{6}\times\tfrac{1}{12}=58\tfrac{4}{12}\text{ ft.}$$
$$\text{or, } \overline{64+20}\times\tfrac{1}{3}\times25\times\tfrac{1}{12}=58\tfrac{4}{12}\text{ ft.}$$

11. The frustrum of a pyramid is 20 ft. × 20 ft. at the base, 8 ft. × 8 ft. at the top, and 45 ft. high. Find the solidity in cubic feet.

$$\left.\begin{array}{r}20^2=400\\ 8^2=\ 64\\ 20\times8=160\end{array}\right\}=624. \qquad \overline{624\div3}\times45=9360\text{ cu. ft.}$$

9360 cubic feet ×·7854 = the contents of a circular frustrum of the same dimensions.

12. The frustrum of a cone measures 8 in. in diameter at the top, and 12 in. in diameter at the butt. Find the mean proportional.

$$8\times12=96.\quad\text{Ans.}$$

The frustrum of a wedge measures 5×3 in. at the top, and 10×6 at the butt. Find the mean proportional.

$$\left.\begin{array}{r}5\times\ 6=30\\ 3\times10=30\end{array}\right\}=60. \qquad 60\div2=30.\quad\text{Ans.}$$

13. The frustrum of a pyramid is 9 ft. square at the base, 4 ft. square at the top, and 12 ft. high. What is the height of the pyramid?

$$\overline{12\times9}\div5=21\tfrac{3}{5}\text{ ft.}\quad\text{Ans.}$$

a. How many cubic feet in a pyramid 21⅗ ft. high, 9 ft. square at the base?

$$\overline{21\tfrac{3}{5}\times9^2}\div3=583\tfrac{1}{5}\text{ ft.}\quad\text{Ans.}$$

b. How many cubic feet in a pyramid 9⅗ ft. high, 4 ft. square at the base?

$$\overline{9\tfrac{3}{5}\times4}\div3=51\tfrac{1}{5}\text{ ft.}\quad\text{Ans.}$$

c. How many cubic feet in the frustrum of a pyramid 9 ft. square at the base. 4 ft. square at the top, and 12 ft. high?

$$9^2+4^2+\overline{9\times4}=133. \qquad 133\times\overline{12\div3}=532\text{ ft.}\quad\text{Ans.}$$

$532+51\frac{1}{5}=583\frac{1}{5}$ cu. ft. = the solidity of the entire pyramid.

14. How many board feet in a telegraph pole 8 in. square at the butt, 5 in. square at the top, and 25 ft. long?

$$\left.\begin{array}{r}8^2=64\\ 5^2=25\\ 8\times5=40\end{array}\right\}=129. \qquad 129\times25\div\overline{3\times12}=89\tfrac{7}{12}\text{ bd. ft.}=7\tfrac{67}{144}\text{ cu. ft.}$$

15. How many board feet in a round pole 8 in. in diameter at base, 5 in. diameter at the top, and 24 ft. long?

$8^2+5^2+8\times5=129. \qquad 129\times24\times{\cdot}0218=67{\cdot}5$ bd. ft.

$129\times24\times{\cdot}001818=5{\cdot}628$ cubic ft.

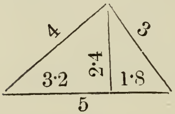
16. Standing on the sea-shore, I see the flash from a ship's gun, and 17 seconds afterwards hear the report. What was the distance of the ship from the shore?

$1125\times17=19{,}125$ ft.

17. What is the weight of the atmosphere on 1 ft. square at the surface of the ocean?

$144\times15=2160$ lbs.

18. The perpendicular let fall on the hypothenuse from the right angle $=\sqrt{}$ of the product of the greater and lesser segment of the hypothenuse; the greater segment $=$ the quotient of the square of the greater side $\div$ the hypothenuse. Thus, the greater segment of the hypothenuse $=4^2\div5=3{\cdot}2$. $3{\cdot}2\times1{\cdot}8=5{\cdot}76$. $\sqrt{5{\cdot}76}=2{\cdot}4=$ the perpendicular let fall.



A man invests £3.600 in the three per cents. @ 90, he sells out at 80, and lends $\frac{5}{8}$ of the proceeds at 4 % and the rest at 5 %. How long must the loan last so that his gain on interest (simple) may exactly equal his loss upon principal?

£3600 @ 90 will buy stock £4000. £3600 − £3200 = Loss = £400
£4000 @ 80 = £3200. $3200\times\frac{3}{8}=1200$. $3200\times\frac{5}{8}=2000$

Interest on £1,200 @ 5 % 1 year = £60
" 2,000 @ 4 % " 80

3 is to 4 as 6 is to 8, then $\frac{4}{7}$ must be earned at 4 %
and $\frac{3}{7}$ " " 5 %

$400\times\frac{4}{7}=£228{\cdot}572. \qquad 400\times\frac{3}{7}=£171{\cdot}429.$

If £1200 earns £60 in 1 year @ 5 %, in what time will it earn £171·429?

If £2000 earns £80 in 1 year @ 4 %, in what time will it earn £228·572?

$20{\cdot}00\times4=80 \mid 228{\cdot}572$ — 2·85715 years.

$12{\cdot}00\times5=60 \mid 171{\cdot}429$ — Years 2·85715

Yrs. 2·85715 = 2 yrs. 312·86 days.

Years 2·85715	28·5715
8	6
Int. @ 4 % = £228·5720	171·4290
Int. @ 5 % = £171·4290	
Total £400·001	

Exercises for Rules on pages 98, 99, &c.

Find the cost of 25 cwts. 2 qrs. 10 lbs. @ 9d. per lb.

$$\begin{array}{r} 2566 \\ 300 \\ \hline 2866 \end{array} \qquad 2866\times\tfrac{3}{4}=2149\tfrac{1}{2}\text{s.}=£107\ 9\text{s.}\ 6\text{d.}$$

Find the cost of 19 cwts. 2 qrs. 18 lbs. @ 7s. 8d. per cwt.
19 cwts. 2 qrs. 18 lbs. @ 1s. per cwt.=19s. 8d. $\times 7\frac{2}{3}$=£7 10s. $9\frac{1}{4}$d.
Find the value of—

1 ton	@ 3d.	per lb.	£$9\frac{1}{3}\times3$=£28 0s. 0d.
,,	@ 5d.	,,	£$9\frac{1}{3}\times5$=£46 13s. 4d.
,,	@ 7d.	,,	£$9\frac{1}{3}\times7$=£65 6s. 8d.
,,	@ $1\frac{1}{4}$d.	,,	£$2\frac{1}{3}\times5$=£11 13s. 4d.
,,	@ $1\frac{3}{4}$d.	,,	£$2\frac{1}{3}\times7$=£16 6s. 8d.
,,	@ $\frac{3}{8}$d.	,,	£$1\frac{1}{6}\times3$= £3 10s. 0d.
,,	@ $\frac{5}{8}$d.	,,	£$1\frac{1}{6}\times5$= £5 16s. 8d.

1 lb.	@ £7	per ton.	$\frac{7}{8}$d. $\times\frac{6}{7}=\frac{6}{8}=\frac{3}{4}$d.
,,	@ £1 15s.	,,	$\frac{1\cdot75}{8}$d. $\times\frac{6}{7}=\frac{1\cdot5}{8}=\frac{3}{16}$d.
,,	@ £3 10s.	,,	$3\frac{1}{2}$d. $\times\frac{6}{7}=\frac{3}{8}$, or, $\frac{3\cdot5}{8}$d. $\times\frac{6}{7}=\frac{3}{8}$d.
,,	@ £21	,,	21 far. $\times\frac{3}{7}$= 9 far.=$2\frac{1}{4}$d.
,,	@ £28	,,	28 far. $\times\frac{3}{7}$=12 far.=3d.
,,	@ £14	,,	·14 far. $\times\frac{3}{7}$= 6 far.=$1\frac{1}{2}$d.
,,	@ £56	,,	56d. $\times\frac{3}{28}$=6d.
,,	@ £84	,,	84d. $\times\frac{3}{28}$=9d.
,,	@ 28s.	per cwt.	28s. $\times\frac{3}{7}$=12 far.=3d.
,,	@ 7s.	,,	7s. $\times\frac{3}{7}$=3 far.
,,	@ 17s. 6d.	,,	$17\frac{1}{2}$ far. $\times\frac{3}{7}=7\frac{1}{2}$ far., or, $17\cdot5\times\frac{3}{7}=7\frac{1}{2}$ far.

112 lbs. @ 3d.=9s. 4d. $\times$3=£1 8s.
112 lbs. @ $\frac{3}{4}$d.=2s. 4d. $\times$3=£0 7s.

To find the cost of one at the following prices per 100.
Write the price per 100 in pence, and point off the two right hand figures.

1 @	10s. 5d. per 100.	125d. ÷ 100=1·25=$1\frac{1}{4}$d.
1 @ £4	1s. 3d. ,,	975d. ÷ 100=9·75=$9\frac{3}{4}$d.
1 @ £1	17s. 6d. ,,	450d. ÷ 100=4·50=$4\frac{1}{2}$d.
1 @ £1	7s. 1d. ,,	325d. ÷ 100=3·25=$3\frac{1}{4}$d.
1 @ £3	0s. 5d. ,,	725d. ÷ 100=7·25=$7\frac{1}{4}$d.

1 sack of flour	@ $2\frac{1}{2}$d.	per lb.	5s. 10d. $\times$10=£2 18s. 4d.
,, ,,	@ $1\frac{3}{4}$d.	,,	5s. 10d. $\times$7=£2 0s. 10d.
1 lb. ,,	@ 630d.	per sack.	630 ÷ 70=9 far.=$2\frac{1}{4}$d.
,, ,,	@ 560d.	,,	560 ÷ 70=8 far.=2d.
,, ,,	@ 490d.	,,	400 ÷ 70=7 far.=$1\frac{3}{4}$d.

Exercises for Rules on pages 100, 101.

Find the cost of 1 oz. @ 5s. per lb. $5 \times 3 = 15$ far. $= 3\frac{3}{4}$d. Ans.
" " 2s. " $2 \times 3 = 6$ " $= 1\frac{1}{2}$d. "
" " 7s. " $7 \times 3 = 21$ " $= 5\frac{1}{4}$d. "
" " 10s. " $10 \times 3 = 30$ " $= 7\frac{1}{2}$d. "
" " 8s. " $8 \times 3 = 24$ " $= 6$d. "

Or the above examples might have been worked as follows:
5s. as 5d. $- \frac{1}{4} = 3\frac{3}{4}$d. 10s. as 10d. $- \frac{1}{4} = 7\frac{1}{2}$d.
2s. as 2d. $- \frac{1}{4} = 1\frac{1}{2}$d. 8s. as 8d. $- \frac{1}{4} = 6$d.
7s. as 7d. $- \frac{1}{4} = 5\frac{1}{4}$d.

Find the cost of 1 lb. at $\frac{3}{4}$d. per oz. 3s. $\div 3 = 1$s.
" " $2\frac{1}{2}$d. " 10s. $\div 3 = 3\frac{1}{3}$s. $= 3$s. 4d.
" " $3\frac{3}{4}$d. " 15s. $\div 3 = 5$s.
" " $6\frac{3}{4}$d. " 27s. $\div 3 = 9$s.
" " $3\frac{1}{4}$d. " 13s. $\div 3 = 4\frac{1}{3}$s. $= 4$s. 4d.

Bought 40 doz. of wine at 78s. per doz. What must each bottle be sold at to gain 20 %? $78 \div 10 = 7{\cdot}8 = 7$s. $9\frac{1}{2}$d.
At 65s. per doz. to gain 20 %? 65s. $\div 10 = 6{\cdot}5 = 6$s. 6d.
" 96s. " " 30 %? 96s. $+ \frac{1}{12} = 104 \div 10 = 10$s. $4\frac{3}{4}$d.
" 64s. " " 35 %? 64s. $+ \frac{1}{8} = 72 \div 10 = 7$s. $2\frac{1}{2}$d.

To gain other rates per cent. see Table, page 101.

6 doz. at $3\frac{1}{2}$d. each. 3s. 6d. $\times 6 =$ £1 1s.
10 " $1\frac{3}{4}$d. " 1s. 9d. $\times 10 =$ 17s. 6d.
4 " $1\frac{1}{4}$d. " 1s. 3d. $\times 4 =$ 5s.
3 " $7\frac{1}{4}$d. " 7s. 3d. $\times 3 =$ £1 1s. 9d.
5 " $4\frac{3}{4}$d. " 4s. 9d. $\times 5 =$ £1 3s. 9d.

If a gross of articles cost 9s., what will 1 cost? $9 \div 3 = \frac{3}{4}$d.
" " 45s. " $45 \div 3 = 15$ far. $= 3\frac{3}{4}$d.
" " 72s. " $72 \div 3 = 24$ far. $= 6$d.
" " 27s. " $27 \div 3 = 9$ far. $= 2\frac{1}{4}$d.
" " 3s. " $3 \div 3 = 1$ farthing.

OPINIONS OF THE PRESS.

" *Court Journal*," *London, 22nd September*, 1888.

"At the Glasgow Exhibition, on the terrace above the switchback railway, a part of the extensive grounds not often visited by tourists, an American, Professor Howard, who has come to this country to revolutionize our system of arithmetic, and sell his own book thereon--at the modest price of one shilling—has erected a platform, from which he orates at intervals, and performs wonders as a lightning calculator. He has instructed a Glasgow boy, named Thomas Freame, aged eleven years, in the Howard method of mental arithmetic, and this not particularly intelligent youth answers questions invited from the bystanders with much rapidity and marvellously correct—such as what is the interest on a large sum of money at any sum per cent. per annum for various broken periods. On what day of the week will a particular date fall any number of years hence, or backwards. A merchant buys a quantity of goods per cwt. for a given sum, at what price per lb. must these goods be retailed to give a profit of 20 per cent. on the transaction, also various rates of profit. Professor Howard then explains on a blackboard how these answers are arrived at by his Anglo-American method, and demonstrates how it is applied to the multiplication or division of several figures, by any number, the answers being given in one line of figures, and correctly."

" *Buteman*," *Rothesay, 22nd September*, 1888.

HOWARD'S ANGLO-AMERICAN ART OF RECKONING.

"This is a manual of arithmetic and general business calculations which should prove invaluable to the trader and all who require rapid and correct methods of reckoning. Mr. Howard gives a number of new and much quicker modes of calculating, for all sorts of businesses, conveyed in a clear and concise manner, and avoiding the technicalities which make books of this class so difficult to comprehend. A number of useful tables of weights, measures, values, &c., are also given. A good proof of the merit of the book is to be found in the fact that it has been adopted as a text-book by the Glasgow School Board."

" *Glasgow Evening News*," *22nd September*, 1888.

"Mr. C. F. Howard, the marvellous calculutor at the Exhibition, has presented Mr. Henry Irving with a copy of his book, and Mr. Irving is studying it with a view to dramatisation."

"*Lanarkshire Examiner*," *22nd September*, 1888.

ARITHMETIC EXTRAORDINARY AT THE EXHIBITION.

"Mr. C. Frusher Howard's lectures and exhibitions on his wonderful manipulations of figures, which take place daily at a covered platform near the Switchback Railway at the Exhibition, have now become very popular as they have got better known, and, as the art having been fully tested, its value has been discovered. That Mr. Howard's art of reckoning has been tried and found a success may be understood, when it is mentioned that the Glasgow School Board has adopted his manual for the commercial arithmetic classes. It does not take long on studying this new method to discover its value, it being as clear and simple, especially for business calculations, as the methods hitherto used in schools are tedious and complex. His rules for reckoning interest, &c., are so clear, so simple, so easily understood as makes one wonder such a method has not been discovered long ere this. How easy, too, Mr. Howard makes it for anyone at all to make up their own interest tables for any number of terms at any rate per cent. There are no difficulties in calculation which Mr. Howard has not tackled, overcome, and made perfectly simple. Not only that, he shows how arithmetic may be made a pleasant recreation for parties of an evening, giving easy rules for making up and working out amusing problems. Then, independent of the instruction to be obtained from Mr. Howard's lectures, it is interesting, and amusing too, to listen to him for a while, for he talks in a very racy manner. But there has been a considerable amount of controversy, and even heated argument in some of the daily journals about Mr. Howard's arithmetic. It is well enough known that there is no race of people on the earth so cautious in adopting any new idea, no matter what it is, until it has been thoroughly tried, than the "canny" Scot. Plenty of examples will be obtained, and free tuition too, by simply listening to Mr. Howard lecturing of an afternoon or evening. Then again, the visitor to the Howard Kiosk may any day witness boys of nine, ten, and eleven years (acquainted with Howard's Anglo-American Art of Reckoning) naming instantly the interest, at various rates per cent., for different periods on any sums of money named by the audience. What other method will enable children of tender years to manipulate figures so rapidly?

"*Glasgow Evening News*," *September*, 1888.

"A feather has just been fixed in Mr. C. Frusher Howard's cap. His system of commercial arithmetic has been adopted by the Glasgow School Board as one of the subjects in the advanced evening classes of the High School. The Anglo-American art of reckoning will, therefore, be taught every Tuesday and Thursday evening to the youth of the city who choose to avail themselves of it."

"*Border Advertiser*," *Galashiels, September 26th*, 1888.

"HOWARD'S ANGLO-AMERICAN ART OF RECKONING.—After a pretty careful examination of the above, we can strongly recommend the work as one of superior merit. It might well supersede some of those text-books now in the hands of pupils in our common schools, which in many cases seem rather a collection of exercises than a guide to the science of arithmetic. Here, there is no long "list of examples" which, as we think, the teacher may easily suggest after the *Rule* has been read, marked, and committed to memory. The method adopted is as follow:—First there is a *definition;* second an *explanation;* third an *example.* Each of these steps can be very easily followed, and they are so intelligently set forth that he who runs may read. But even more as an aid to mental arithmetic will this work be of value. The Rules for Farmers, Mechanics, &c., are so tabulated that, with a little practice, the number of acres in a field, or the number of solid feet in a round log, may be easily ascertained. Reference Tables on such subjects as "Specific Gravity," "Latitude and Longitude," "Metric System," are arranged so distinctly that the eye can catch the information required just at a glance. A most ingenious "Perpetual Calender" brings to a close one of the best Arithmetical Handbooks one can possess. Young lads who have just *passed* all the standards of the Day School should supply themselves with this book. It is really what it claims to be—"Exact! Clear! Brilliant!"

"*Glasgow Mail*," *5th September*, 1888.

ARITHMETIC AT THE EXHIBITION.

"The number of boys who are daily to be seen at Mr. Howard's stand studying his system of arithmetic, and frequently taking part in the manipulation of figures before large audiences, is proof sufficient that in the Anglo-American Art of Reckoning they have found an object of very great interest to them to which they look for facilitating their school work, and fitting them as rapid calculators for commercial life in the near future."

"*Glasgow Evening News.*"

"Every day visitors to the Exhibition are always on the look-out for something new to attract their attention. Let them only take a walk of an evening, between the hours of six and nine, on the south side of the Kelvin past the Bishop's Castle, and they will assuredly find something to amuse and entertain them in Mr. Howard's Anglo-American Art of Reckoning. It is as much an entertainment as an exhibit—as much an exhibit as an entertainment. It is also in all truth what it is designated—a most marvellous manipulation of figures."

"*Hamilton Advertiser,*" *22nd September,* 1888.

HOWARD'S ANGLO-AMERICAN ART OF RECKONING.

"We have received a copy of this useful, educational *brochure,* which is fast taking its place as a text book in the leading educational institutions of the country. From the most elementary stages, the student is carried, within the limits of 126 pages, to the most advanced and complete phases of the art. The Yankee directness and clearness are conspicuous throughout; and while this explains the position to which it has already attained, it leaves no doubt as to the place it is calculated to fill in the future."

"*Coatbridge Express,*" *19th September,* 1888.

HOWARD'S ANGLO-AMERICAN ART OF RECKONING.

"One of the most practical novelties at present to be witnessed at the Glasgow International Exhibition, and which is likely to prove of material benefit to the commercial community is Howard's Anglo-American Art of Reckoning. In a covered stall, a little beyond the Bishop's Palace, Mr. Howard gives demonstrations of his novel system of practical arithmetic, which we heartily commend to all young men, having listened to his practical lecture the other day with much interest. One of the chief advantages of the system is that its practical application can be readily acquired even by those whose natural capacities or educational opportunities may not be such as to make them masters of principles and theories. The book treats of common and decimal fractions in an intelligent and intelligible style, while the rapid rules for computing interest—simple and compound—for squaring numbers, for discount, exchange, the equation of payments, etc., etc., possess a novel and special excellence that will be at once appreciated by all who are at all conservant with the ordinary methods. We observe with pleasure that the Glasgow School Board has adopted Mr. Howard's Book for the teaching of pupils in the commercial arithmetic classes for the evening schools in the coming session."

"*The Wishaw Press,*" *Wishaw, 22nd September,* 1888.

"QUICK AT FIGURES."

"There is no royal road to learning, but the difficulties can occasionally be bridged over by artificial helps. A book entitled 'Howard's Anglo-American Art of Reckoning' meets the requirements of all who wish to be 'quick at figures.' It treats of common and decimal fractions in an intelligent and intelligible style, while the rapid rules for computing interest—simple and compound, for squaring numbers, for discount, exchange, the equation of payments, etc., etc., possess a novel and special excellence that will be at once appreciated by any one who is at all conversant with the ordinary methods. We notice that the Glasgow School Board has adopted Mr. Howard's book for the teaching of pupils in the commercial arithmetic classes for the evening schools in the coming session, and the practical demonstrations of his novel system at present form one of the attractions of the Glasgow Exhibition."

"*Kirkintilloch Herald,*" *19th September,* 1888.

HOWARD'S ANGLO-AMERICAN ART OF RECKONING.

"Of the attractive and interesting specialties in the Glasgow Exhibition Howard's arithmetic demonstrations have received a considerable amount of attention, and few of those who have listened to Mr. Howard's expositions have gone away without taking a copy of his book with them. The success has been phenomenal, and no less than 360,000 copies have been sold since the first issue of the book. We observe it has been adopted as a text book in the Glasgow Board Schools."

"*Invergordon Times,*" *22nd September,* 1888.

"QUICKNESS AT FIGURES."

"Mr. C. Frusher Howard, whose book entitled "Howard's Anglo-American Art of Reckoning" has obtained such popularity among all classes of people desirous of emulating the performances of the "lightning calculators," has issued another edition of his work. The great feature of the book is that it throws aside all needless work in calculation and goes straight for the solution of each question. In some cases there is nothing for it but hard and persevering work, but in a great number of instances the drudgery of calculation can be transformed into pleasing pastime. The book is published by C. Frusher Howard, American Exchange, 449, Strand, London."

"*Arbroath Guide*," 22*nd September*, 1888.

HOWARD'S ANGLO-AMERICAN ART OF RECKONING.

"The 'Anglo-American Art of Reckoning' is a most interesting book of Arithmetic. It is described on the title page as 'the standard teacher and referee of shorthand business arithmetic,' and the description is accurate. The book is designed for the use of schools and business colleges, as well as to be a manual for the counting-house and self-culture. The rules are of the simplest, and are all framed with a view to rapidity as well as accuracy in calculation. Mr. Howard's work is one of the text-books in use in the schools under the Glasgow School Board for commercial arithmetic. It received the highest award of merit in the American Exhibition in London last year, and the exposition of its system has attracted very considerable attention in the Glasgow Exhibition of this year. It can be confidently recommended to schools, and for private use, as a most useful book."

"*The Ross-shire Journal*," 21*st September*, 1888.

CALCULATING EXTRAORDINARY.

"Some time ago we published a paragraph detailing the marvellous calculating feats of a local porter, in which results were arrived at without apparent "time or trouble." Mr. C. Frusher Howard has just issued a new edition of his Anglo-American art of reckoning, by which business calculations are made with astonishing ease, accuracy, and speed. The 'needless' work in our ordinary school-book rules are omitted, and problems are solved by an easily-learned, simple, and natural arrangement. In short the new method of calculating enables persons of ordinary intellect to surpass the performances of the 'lightning calculators' who have astonished mankind. The book, we understand, is the text book for arithmetic, used in the advanced evening classes held in the High School of Glasgow, under the School Board of that city. We learn that Mr. Howard has a stand at the exhibition, where he has several young lads who have acquired his system of calculation, and who, by that system, can perform feats in calculating which are simply marvellous."

Howard's Copyright Perpetual Calendar.

5	4	3	2	1	7	6
00	01	02	03	*	04	05
06	07	……	08	09	10	11
……	12	13	14	15	……	16
17	18	19	……	20	21	22
23	……	24	25	26	27	……
28	29	30	31	……	32	33
34	35	……	36	37	38	39
……	40	41	42	43	……	44
45	46	47	……	48	49	50
51	……	52	53	54	55	……
56	57	58	59	……	60	61
62	63	……	64	65	66	67
……	68	69	70	71	……	72
73	74	75	……	76	77	78
79	……	80	81	82	83	……
84	85	86	87	……	88	89
90	91	……	92	93	94	95
	96	97	98	99		

Index Figures for the years 1800 to 1899.

Take the index figure above the given year; in a line with the given month, find the corresponding figure; above are the days of the month and week.

1 S.	1 Sa	1 F.	1 Th	1 W.	1 Tu	1 M.
2 M.	2 S.	2 Sa	2 F.	2 Th	2 W.	2 Tu
3 Tu	3 M.	3 S.	3 Sa	3 F.	3 Th	3 W.
4 W.	4 Tu	4 M.	4 S.	4 Sa	4 F.	4 Th
5 Th	5 W.	5 Tu	5 M.	5 S.	5 Sa	5 F.
6 F.	6 Th	6 W.	6 Tu	6 M.	6 S.	6 Sa
7 Sa	7 F.	7 Th	7 W.	7 Tu	7 M.	7 S.
8 S.	8 Sa.	8 F.	8 Th	8 W.	8 Tu	8 M.
9 M.	9 S.	9 Sa	9 F.	9 Th	9 W.	9 Tu
10 Tu	10 M.	10 S.	10 Sa	10 F.	10 Th	10 W.
11 W.	11 Tu	11 M.	11 S.	11 Sa	11 F.	11 Th
12 Th	12 W.	12 Tu	12 M.	12 S.	12 Sa	12 F.
13 F.	13 Th	13 W.	13 Tu	13 M.	13 S.	13 Sa
14 Sa	14 F.	14 Th	14 W.	14 Tu	14 M.	14 S.
15 S.	15 Sa.	15 F.	15 Th	15 W.	15 Tu	15 M.
16 M.	16 S.	16 Sa	16 F.	16 Th	16 W,	16 Tu
17 Tu	17 M.	17 S.	17 Sa	17 F.	17 Th	17 W.
18 W.	18 Tu	18 M.	18 S.	18 Sa	18 F.	18 Th
19 Fh	19 W.	19 Tu	19 M.	19 S.	19 Sa	19 F.
20 F.	20 Th	20 W.	20 Tu	20 M.	20 S.	20 Sa
21 Sa.	21 F.	21 Th	21 W.	21 Tu	21 M.	21 S.
22 S.	22 Sa	22 F.	22 Th	22 W.	22 Tu	22 M.
23 M.	23 S.	23 Sa	23 F.	23 Th	23 W.	23 Tu
24 Tu	24 M.	24 S	24 Sa	24 F.	24 Th	24 W.
25 W.	25 Tu	25 M.	25 S.	25 Sa	25 F.	25 Th
26 Th	26 W.	26 Tu	26 M.	26 S.	26 Sa	26 F.
27 F.	27 Th	27 W.	27 Tu	27 M.	27 S.	27 Sa
28 Sa.	28 F.	28 Th	28 W.	28 Tu	28 M.	28 S.
29 S.	29 Sa.	29 F.	29 Th	29 W.	29 Tu	29 M.
30 M.	30 S.	30 Sa	30 F.	30 Th	30 W.	30 Tu
31 Tu	31 M.	31 S.	31 Sa	31 F.	31 Th	31 W.

January	1	2	3	4	5	6	7
February	4	5	6	7	1	2	3
March	4	5	6	7	1	2	3
April	7	1	2	3	4	5	6
May	2	3	4	5	6	7	1
June	5	6	7	1	2	3	4
July	7	1	2	3	4	5	6
August	3	4	5	6	7	1	2
September	6	7	1	2	3	4	5
October	1	2	3	4	5	6	7
November	4	5	6	7	1	2	3
December	6	7	1	2	3	4	5

3, 2, 1, 7, 6, 5, 4, 18th Cent'y. Index 20th Cent'y, 7, 6, 5, 4, 3, 2, 1.

* THE LEAP YEARS FOLLOW THE BLANK SPACES: FOR THESE YEARS USE THE INDEX FIGURES ABOVE THE BLANK SPACES FOR JAN. AND FEB., OR COUNT ONE DAY BACK.